The Limitations of Phase Reduction Techniques – A Case Study of the Morris-Lecar Model

Stephen Coombes

School of Mathematical Sciences, The University of Nottingham, UK

A common approach in computational neuroscience is to assume that a single neuron model with a strongly attracting limit-cycle can be described solely by its phase on cycle. Given the transform from the original phase space to the circle one can then build networks of phase-oscillators. This makes sense if the coupling is weak (so that limit-cycles persist) - and one may use the infinitesimal Phase Response Curve (iPRC) for the uncoupled neuron to build network interactions. However, the main limitation of all of this is the assumption of 'phase' - and that motion away from the limit cycle is ignored. In this talk I will show that for the widely used Morris-Lecar model this standard perturbative technique gives different predictions than the full model. In particular shear-induced chaos, i.e., chaotic behaviour that results from the amplification of small perturbations by underlying shear, is missed entirely by the iPRC. To address this issue I will discuss the use of phase-amplitude coordinates that allow one to track the evolution of distance from the cycle as well as phase on the cycle. I will illustrate how this can provide a better description of single neuron response to both periodic and stochastic forcing. I will also show how it paves the way for a better understanding of network dynamics, and is particularly relevant to understanding the large amplitude bursting oscillations (in the mean membrane potential) seen in networks of gap-junction coupled Morris-Lecar networks.