

Interface Dynamics in Planar Neural Field Models

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Neural field models describe the coarse-grained activity of populations of interacting neurons. Because of the laminar structure of real cortical tissue they are often studied in two spatial dimensions, where they are well known to generate rich patterns of spatiotemporal activity. Such patterns have been interpreted in a variety of contexts ranging from the understanding of visual hallucinations to the generation of electroencephalographic signals. Typical patterns include localised solutions in the form of travelling spots, as well as intricate labyrinthine structures. These patterns are naturally defined by the interface between low and high states of neural activity. Here we derive the equations of motion for such interfaces and show, for a Heaviside firing rate, that the normal velocity of an interface is given in terms of a non-local Biot-Savart type interaction over the boundaries of the high activity regions. This exact, but dimensionally reduced, system of equations is solved numerically and shown to be in excellent agreement with the full nonlinear integral equation defining the neural field. We develop a linear stability analysis for the interface dynamics that allows us to understand the mechanisms of pattern formation that arise from instabilities of spots, rings, stripes and fronts. We further show how to analyse neural field models with linear adaptation currents, and determine the conditions for the dynamic instability of spots that can give rise to breathers and travelling waves. We end with a discussion of amplitude equations for analysing behaviour in the vicinity of a bifurcation point (for smooth firing rates). The condition for a drift instability is derived and a center manifold reduction is used to describe a slowly moving spot in the vicinity of this bifurcation. This analysis is extended to cover the case of two slowly moving spots, and establishes that these will reflect from each other in a head-on collision.