



Explaining and Exploiting Impedance Modulation in Motor Control

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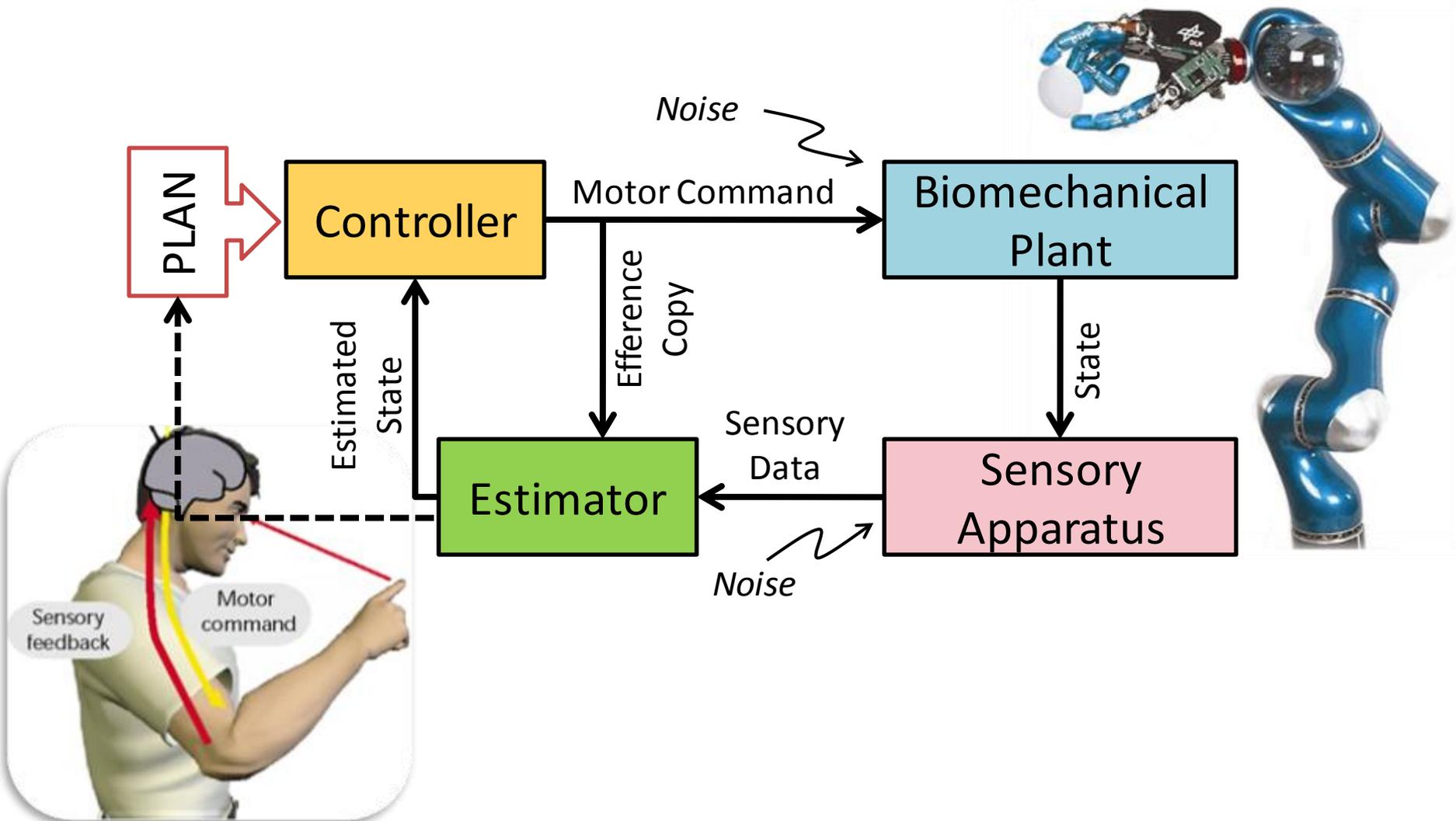
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Director, **Edinburgh Centre for Robotics**

www.edinburgh-robotics.org



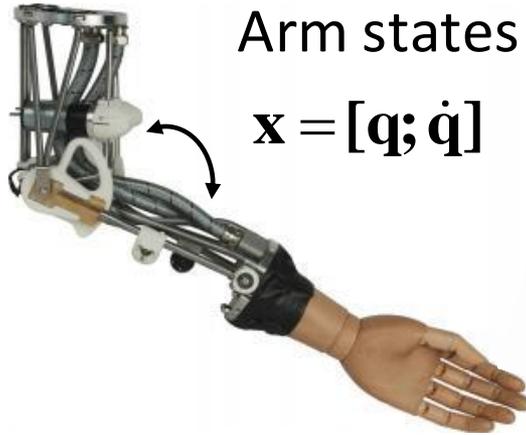
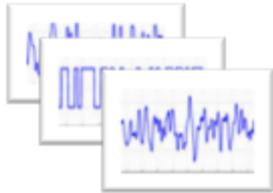
Sensorimotor Control



Planning with Redundancy

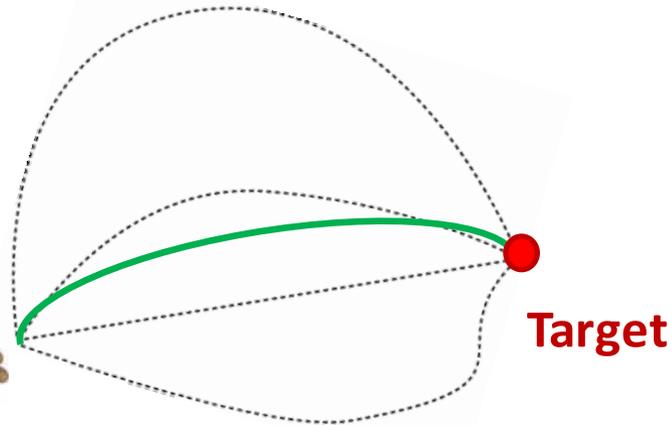
Control signals

\mathbf{u}



Arm states

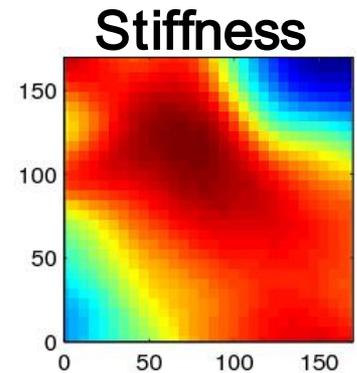
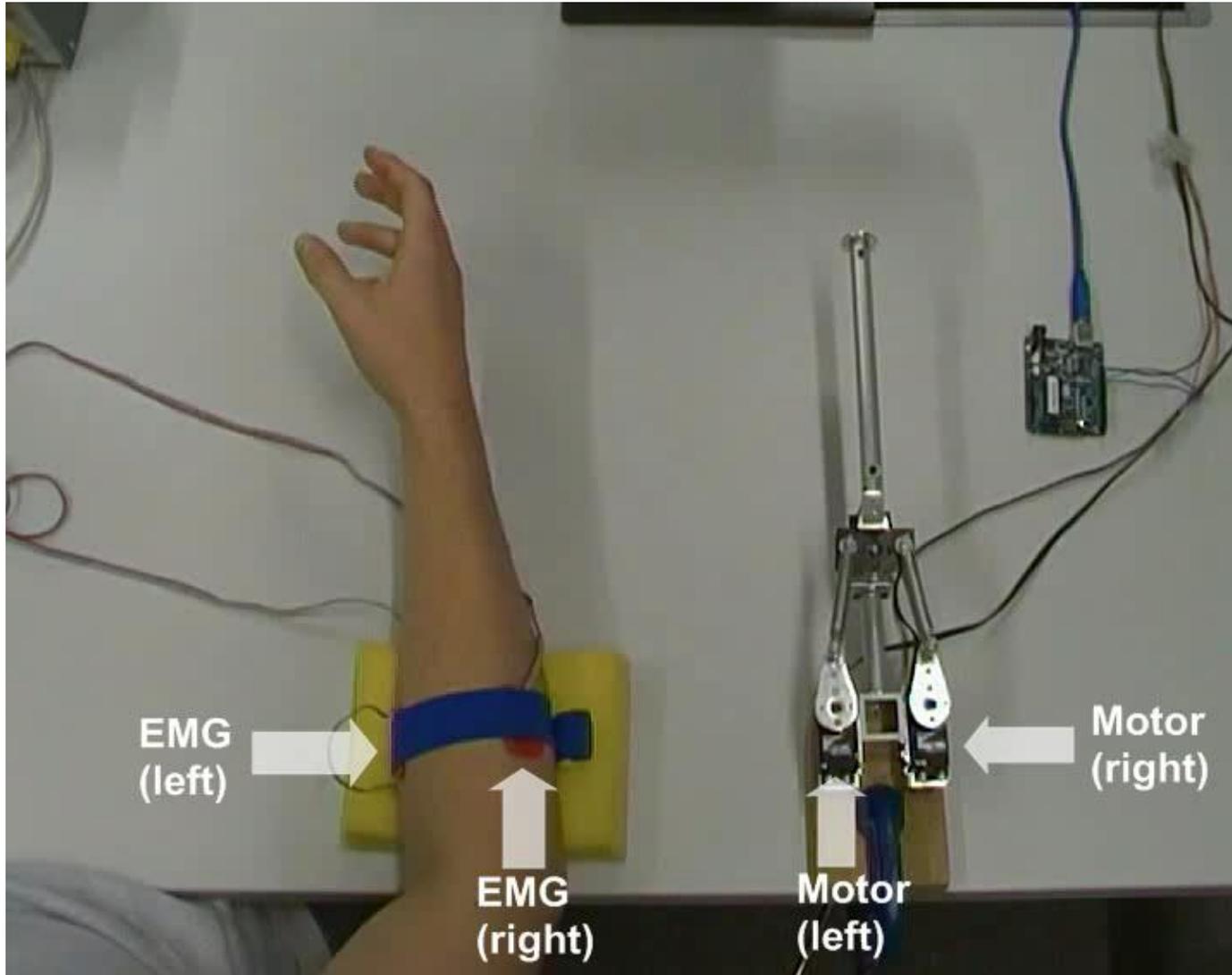
$\mathbf{x} = [\mathbf{q}; \dot{\mathbf{q}}]$



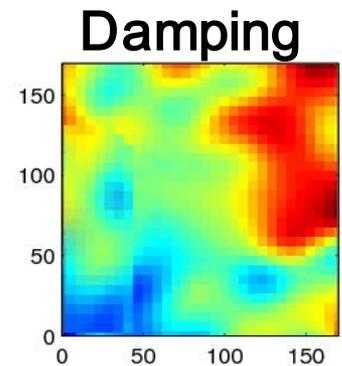
Redundancy at various levels:

- Task -> End Effector Trajectory (*Min. Jerk, Min. Energy etc.*)
- End Effector -> Joint Angles (*Inverse Kinematics*)
- Joint Angles -> Joint Torques (*Inverse Dynamics*)
- Joint Torques -> Joint Stiffness (*Variable Impedance*)

Variable Stiffness Actuation



+



Impedance

Compliant Behaviour on a Complex Anthropomorphic Arm

This capability is crucial for **safe, yet precise** human robot interactions and **wearable exoskeletons**.

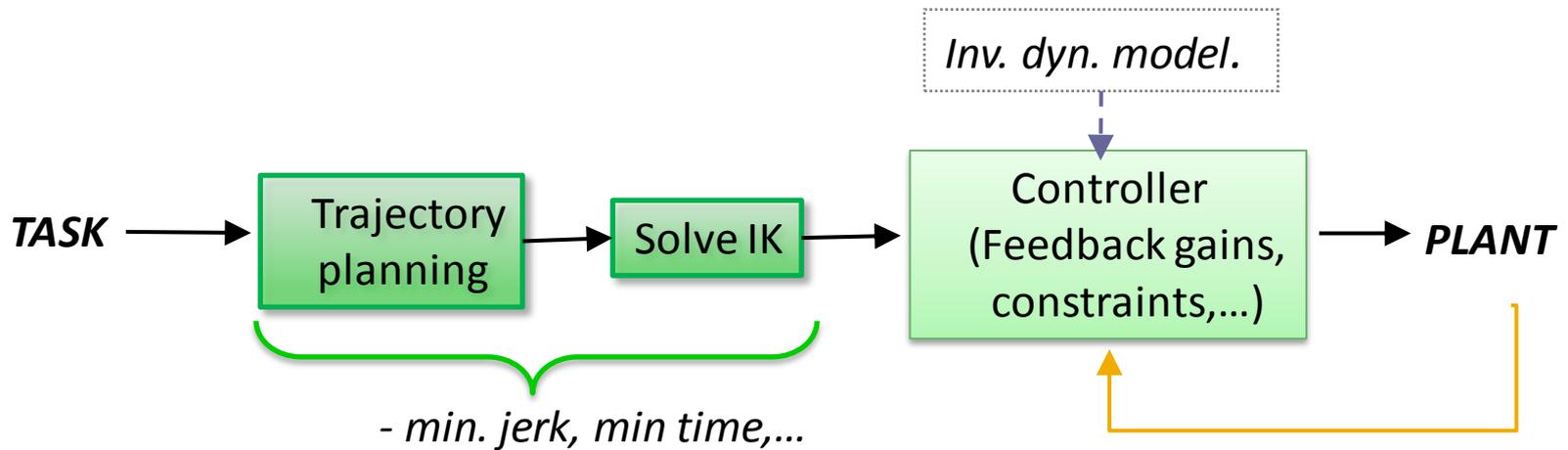
HAL Exoskeleton, Cyberdyne Inc., Japan



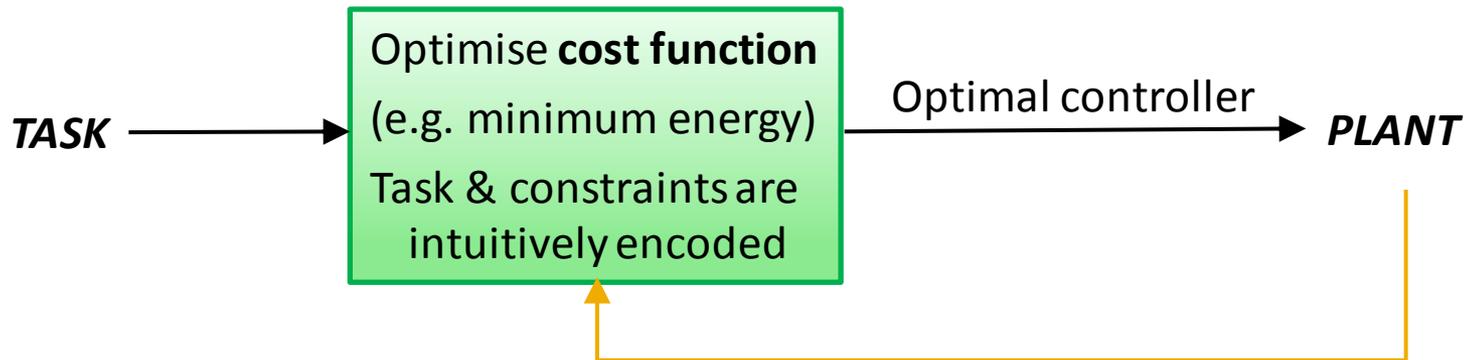
KUKA 7 DOF arm with Schunk 7 DOF hand @ Univ. of Edinburgh

Plan Optimization and Control

Open Loop OC



OFC



Optimal Feedback Control

Given:

- Start & end states,
- fixed-time horizon T and
- system dynamics $d\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u})dt + \mathbf{F}(\mathbf{x}, \mathbf{u})d\omega$

How the system reacts ($\Delta\mathbf{x}$) to forces (\mathbf{u})

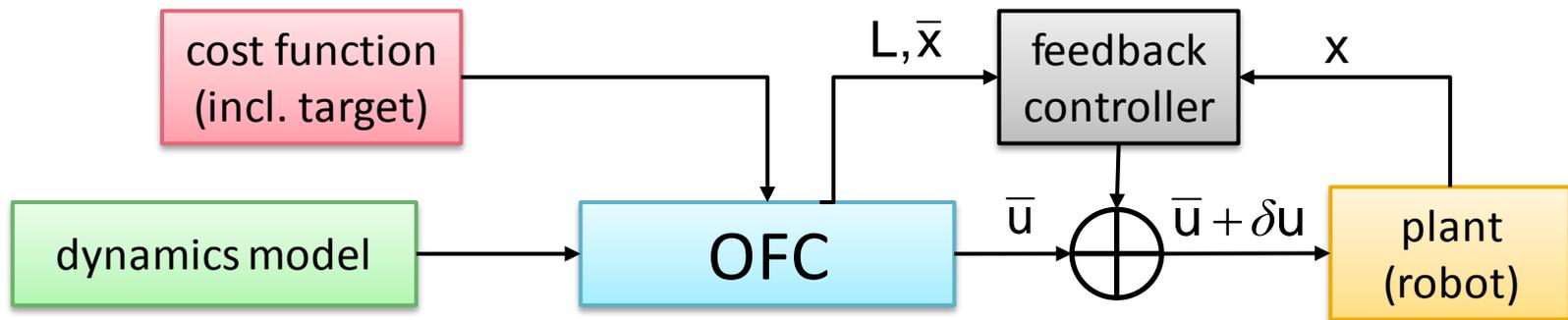
And assuming some cost function:

$$v^\pi(t, \mathbf{x}) \equiv E \left[\underbrace{h(\mathbf{x}(T))}_{\text{Final Cost}} + \underbrace{\int_t^T l(\tau, \mathbf{x}(\tau), \boldsymbol{\pi}(\tau, \mathbf{x}(\tau)))d\tau}_{\text{Running Cost}} \right]$$

Apply **Statistical Optimization** techniques to find optimal **control commands**

Aim: find control law π^* that minimizes $v^{\pi^*}(0, \mathbf{x}_0)$.

What does an OFC generate?



OFC law

$$\mathbf{u}_k^{plant} = \bar{\mathbf{u}}_k + \delta \mathbf{u}_k$$
$$\delta \mathbf{u}_k = \mathbf{L}_k \cdot (\mathbf{x}_k - \bar{\mathbf{x}}_k)$$

Choice of Optimization Methods

- Analytic Methods
 - Linear Quadratic Regulator (LQR)
 - Linear Quadratic Gaussian (LQG)
- Local Iterative Methods
 - iLQG, iLDP
- Dynamic Programming (DDP)
- Inference based methods
 - AICO, PI², ψ -Learning

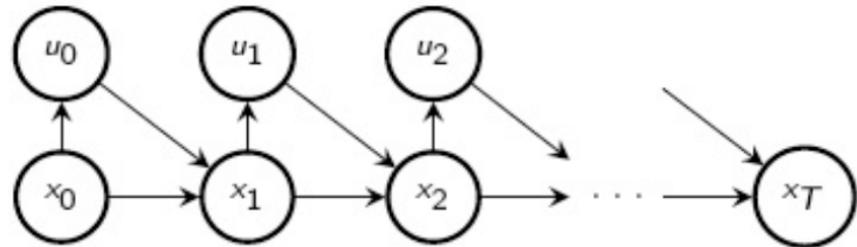
SOC through Approximate Inference

Given:

- ▶ Discrete time controlled stochastic process

State: $x_t \in \mathbb{X} = \mathbb{R}^n$
 $\bar{x} = (x_0, \dots, x_T)$

Control: $u_t \in \mathbb{U} = \mathbb{R}^m$
 $\bar{u} = (u_0, \dots, u_T)$



Transition Probability:

$P(x_{t+1}|x_t, u_t)$ (typically $P(x_{t+1}|x_t, u_t) = \mathcal{N}(x_{t+1}; f(x_t, u_t), \mathbf{Q})$)

- ▶ Cost function

$$\mathcal{C}(\bar{x}, \bar{u}) = \sum_{t=0}^T \mathcal{C}_t(x_t, u_t) \quad \mathcal{C}_t(\cdot, \cdot) \geq 0$$

Solve:

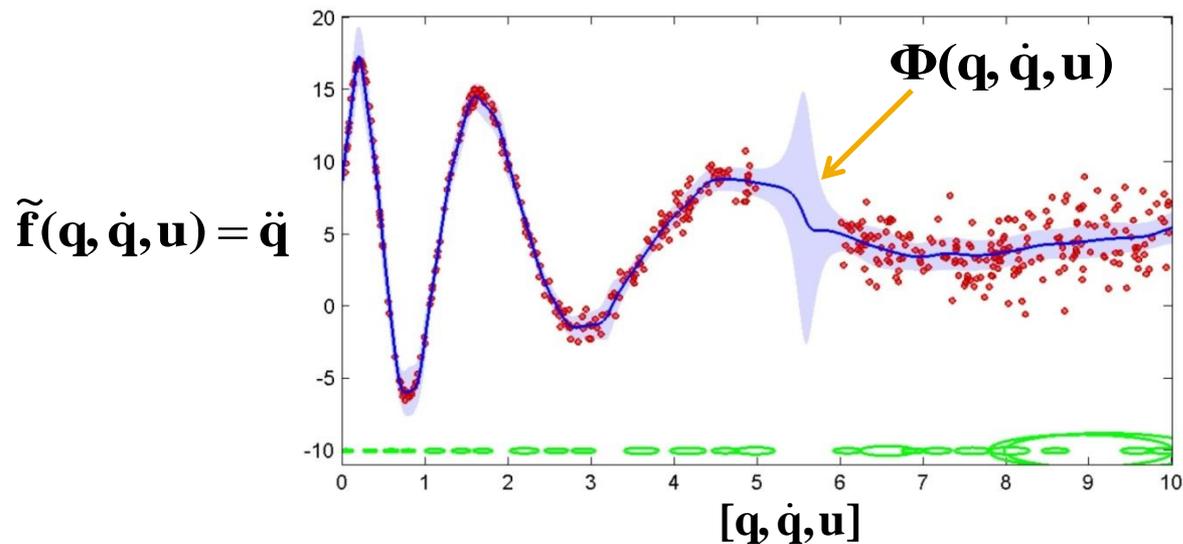
$$\pi^* = \operatorname{argmin}_{\pi} \langle \mathcal{C}(\bar{x}, \bar{u}) \rangle_{\bar{x}, \bar{u} | x_0, \pi}$$

Optimal Variable Impedance

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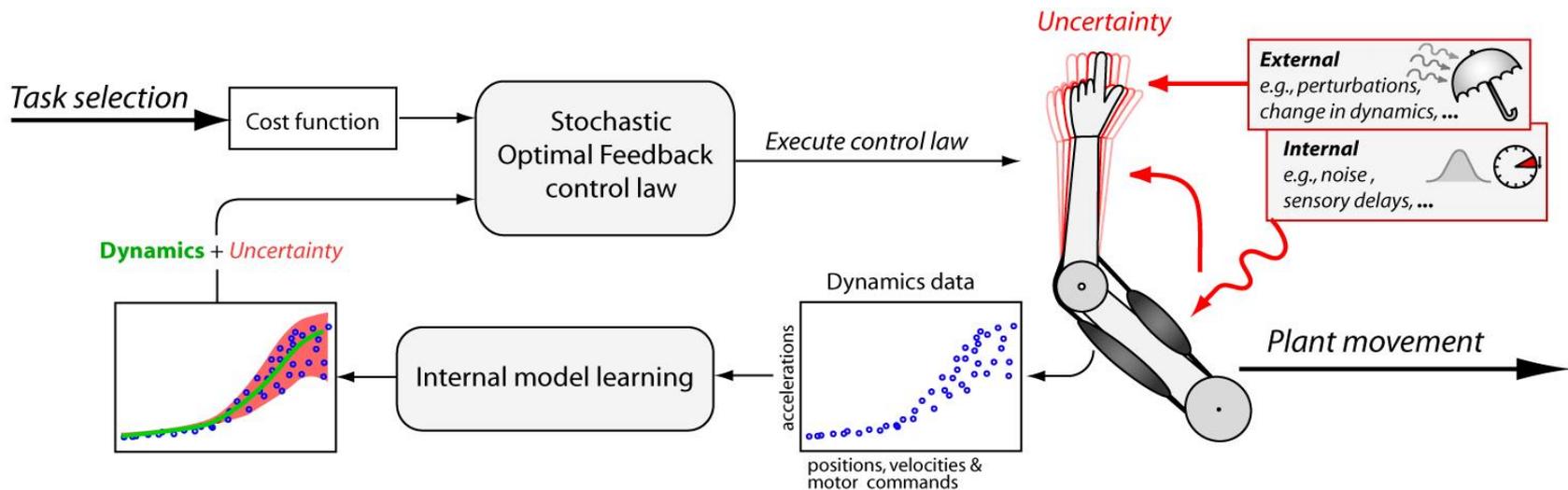
Dynamics Learning with LWPR

Locally Weighted Projection Regression (LWPR) for dynamics learning
(Vijayakumar et al., 2005).



$$d\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u})dt + \mathbf{F}(\mathbf{x}, \mathbf{u})d\omega \quad \longrightarrow \quad d\mathbf{x} = \tilde{\mathbf{f}}(\mathbf{x}, \mathbf{u})dt + \phi(\mathbf{x}, \mathbf{u})d\omega$$

OFC with Learned Dynamics (OFC-LD)



- OFC-LD uses LWPR learned dynamics for optimization (Mitrovic et al., 2010a)
- Key ingredient: Ability to learn both the dynamics and the **associated uncertainty** (Mitrovic et al., 2010b)

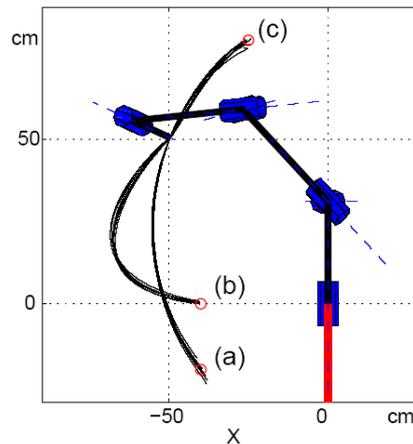
OFC-LD: Advantages

Reproduces the “trial-to-trial” variability in the uncontrolled manifold, i.e., exhibits the **minimum intervention principle** that is characteristic of human motor control.

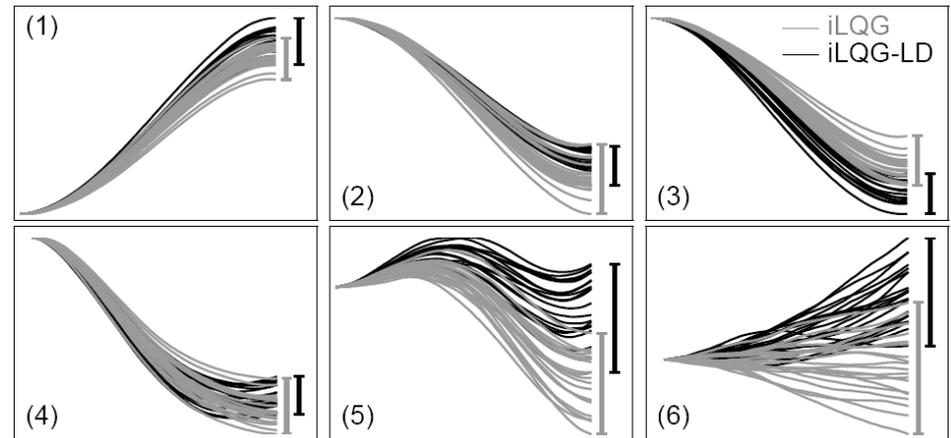
KUKA LWR



Simulink Model



Minimum intervention principle



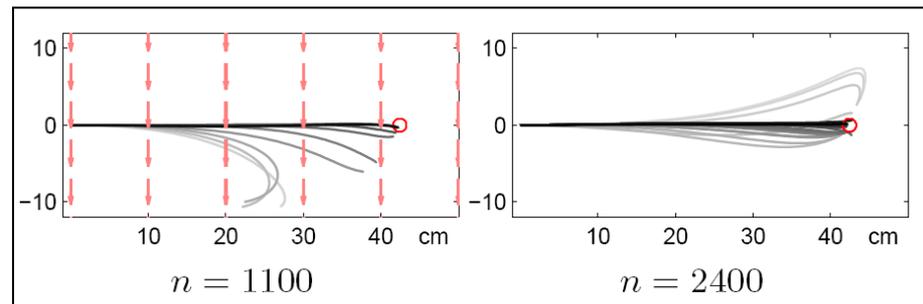
OFC-LD: Explaining Motor Adaptation

Can **predict** the “ideal observer” **adaptation behaviour** under complex force fields due to the ability to work with adaptive dynamics

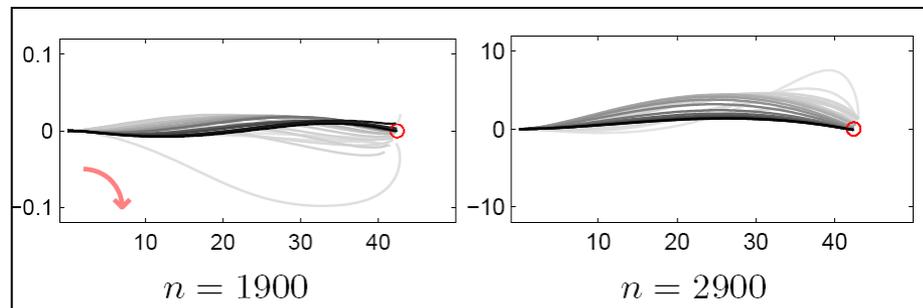
Cost Function:

$$v = w_p |\mathbf{q}_K - \mathbf{q}_{tar}|^2 + w_v |\dot{\mathbf{q}}_K|^2 + w_e \sum_{k=0}^K |\mathbf{u}_k|^2 \Delta t.$$

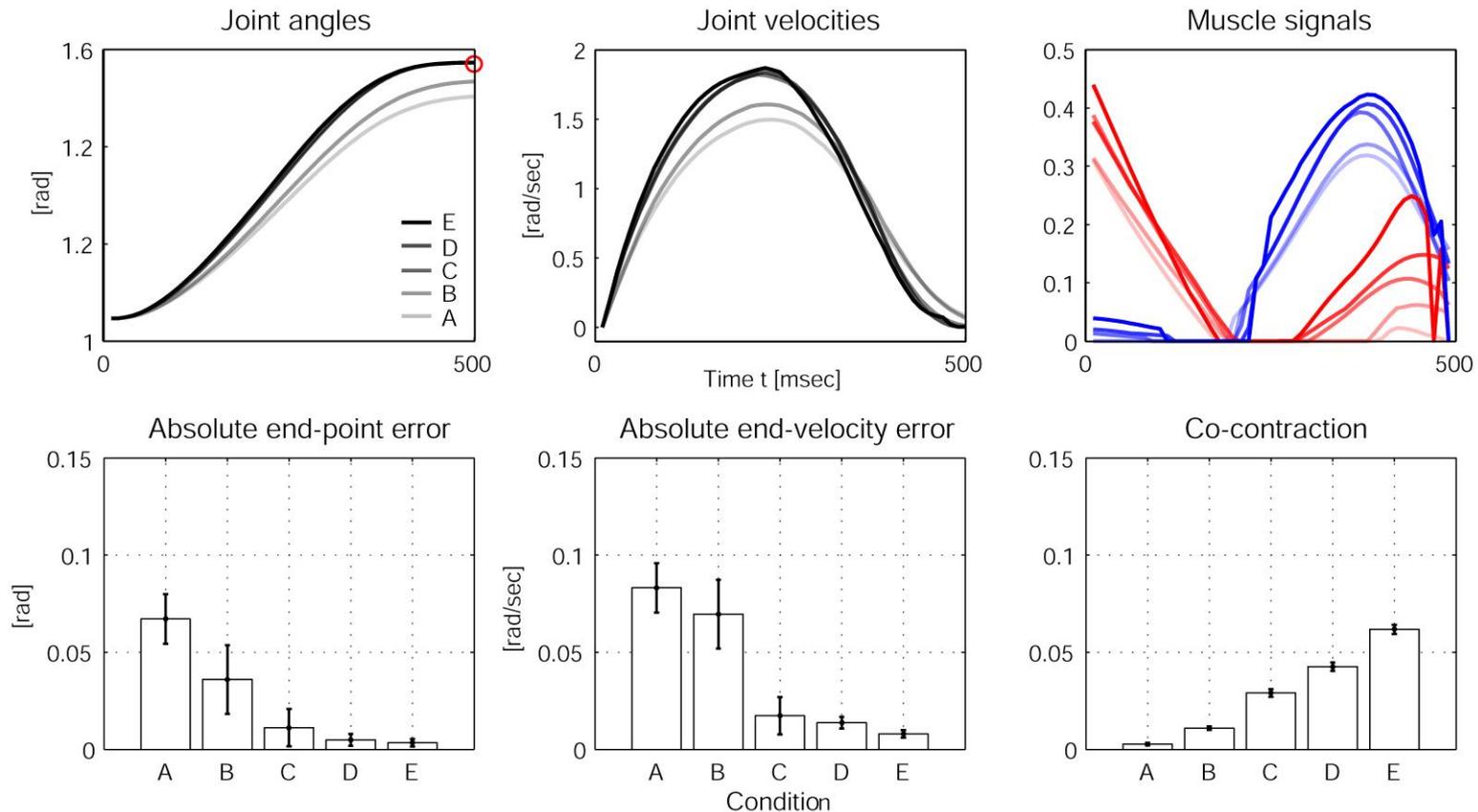
Constant Unidirectional Force Field



Velocity-dependent Divergent Force Field



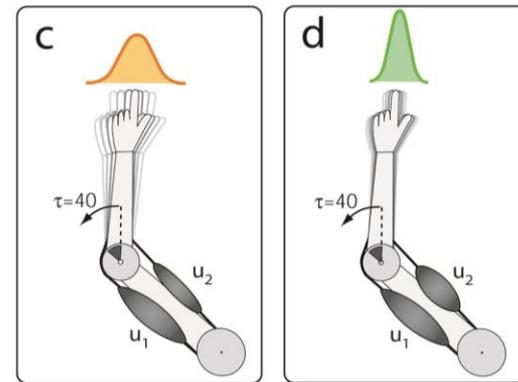
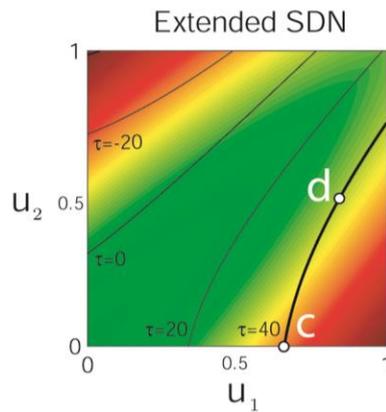
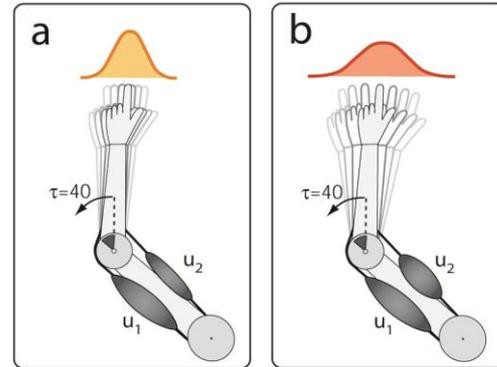
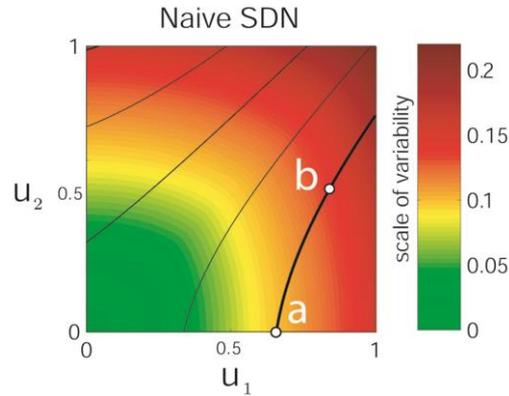
Results: Higher accuracy demands



See: Osu et al., 2004; Gribble et al., 2003

Realistic kinematic variability

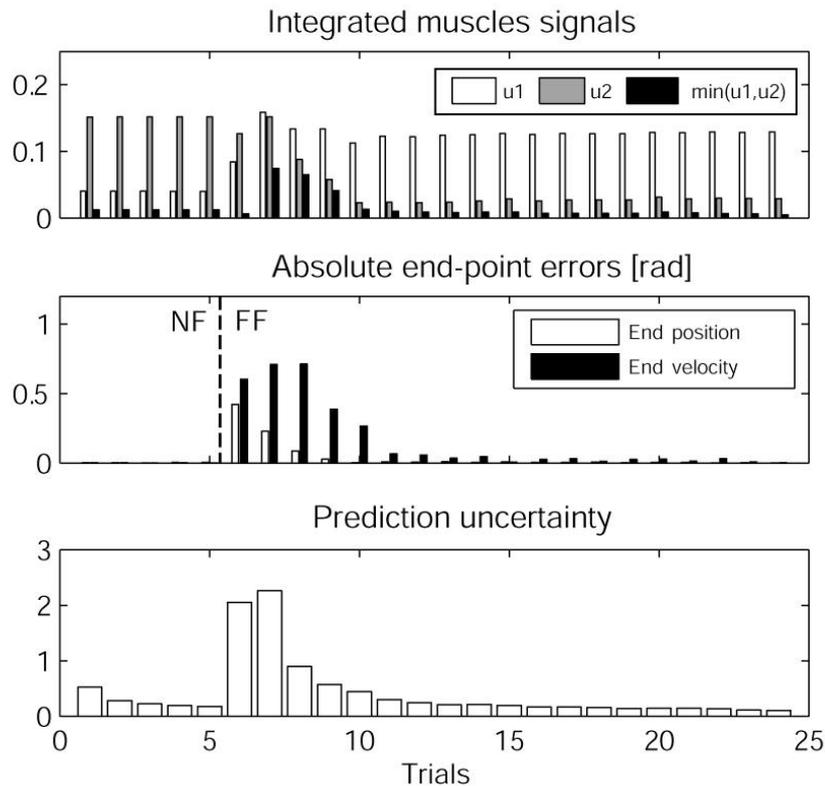
Focus: Signal Dependent Noise (SDN)



$$\sigma(\mathbf{u}) = \sigma_{isotonic} |u_1 - u_2|^n + \sigma_{isometric} |u_1 + u_2|^m, \quad \xi \sim N(0, \mathbf{I}_2)$$

Results: Adaptation to external force fields

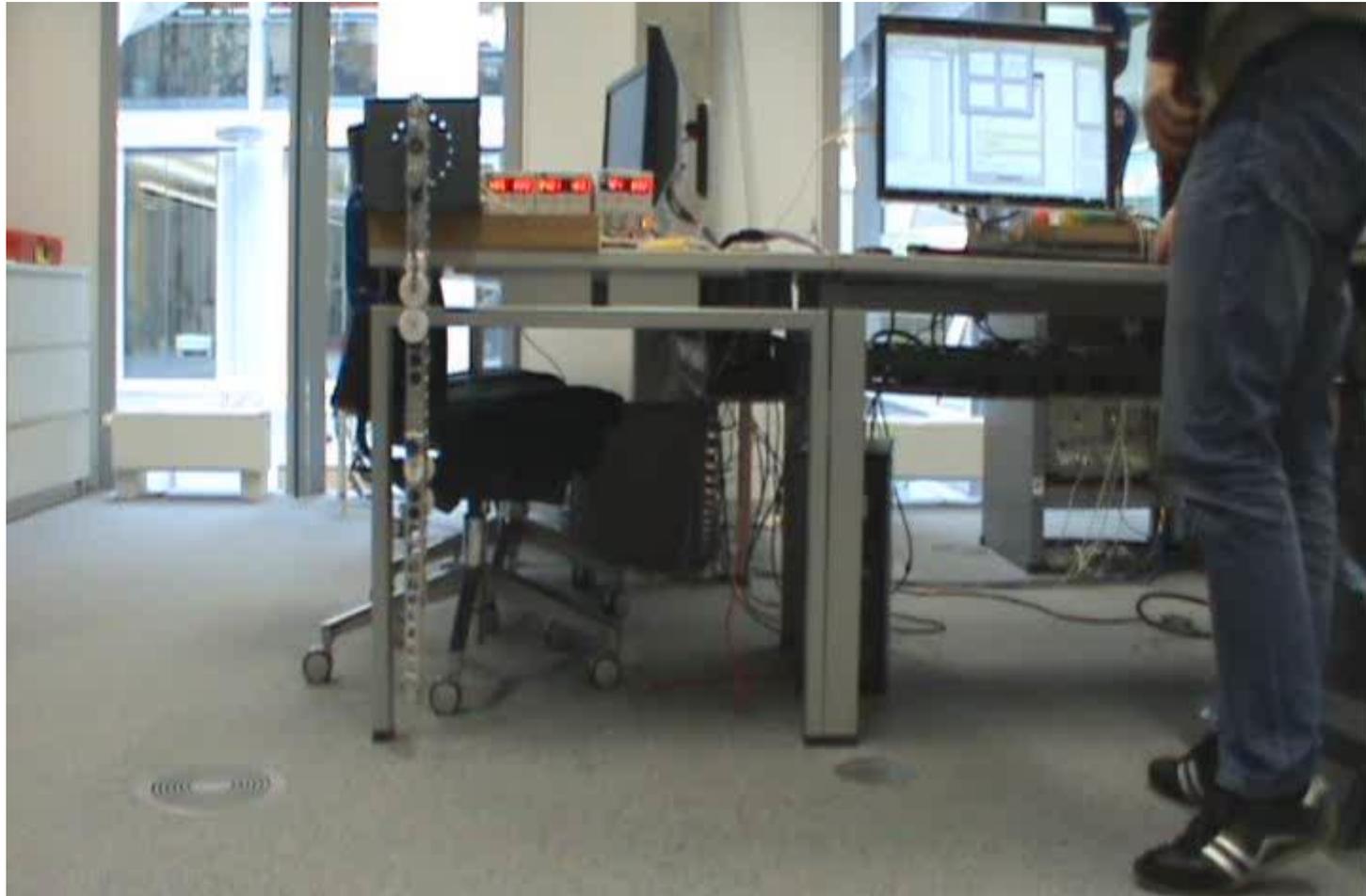
Stochastic OFC-LD



Optimal Variable Impedance

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 - Explosive Movement Tasks (e.g., throwing)
 - Periodic Movement Tasks and Temporal Optimization (e.g. walking, brachiation)

Highly dynamic tasks, explosive movements



David Braun, Matthew Howard and Sethu Vijayakumar, Exploiting Variable Stiffness for Explosive Movement Tasks, *Proc. Robotics: Science and Systems (R:SS)*, Los Angeles (2011)

The two main ingredients:

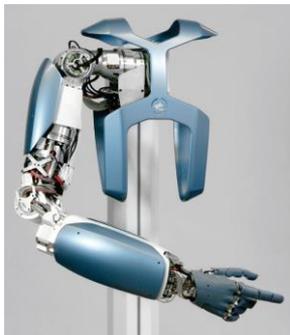
Compliant Actuators

Torque/Stiffness Opt.

- VARIABLE JOINT STIFFNESS



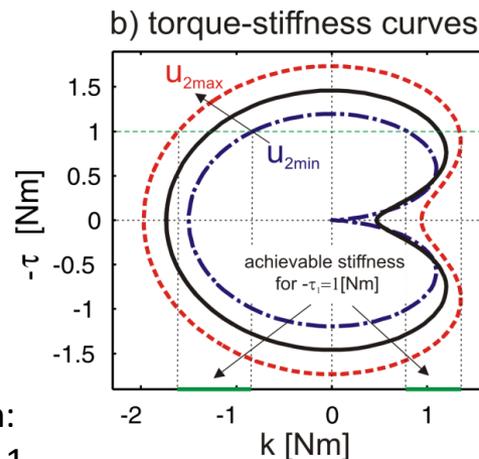
MACCEPA:
Van Ham et.al, 2007



DLR Hand Arm System:
Grebenstein et.al., 2011

$$\boldsymbol{\tau} = \boldsymbol{\tau}(\mathbf{q}, \mathbf{u})$$

$$\mathbf{K} = \mathbf{K}(\mathbf{q}, \mathbf{u})$$



- Model of the system dynamics:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad \mathbf{u} \in \Omega$$

- Control objective:

$$J = -d + w \frac{1}{2} \int_0^T \|\mathbf{F}\|^2 dt \rightarrow \min.$$

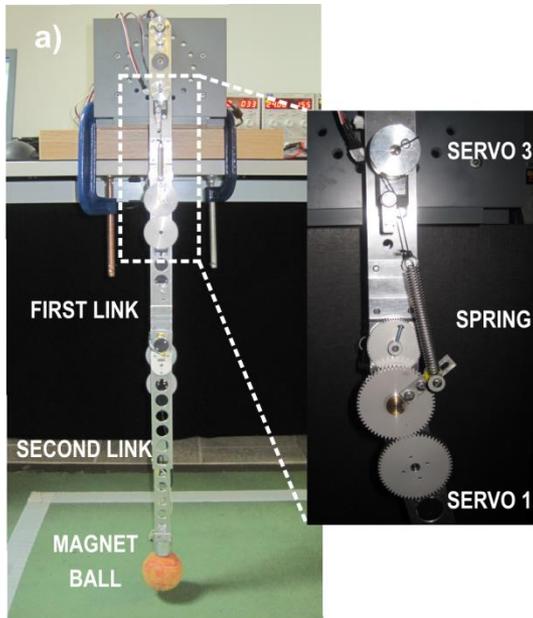
- Optimal control solution:

$$\mathbf{u}(t, \mathbf{x}) = \mathbf{u}^*(t) + \mathbf{L}^*(t)(\mathbf{x} - \mathbf{x}^*(t))$$

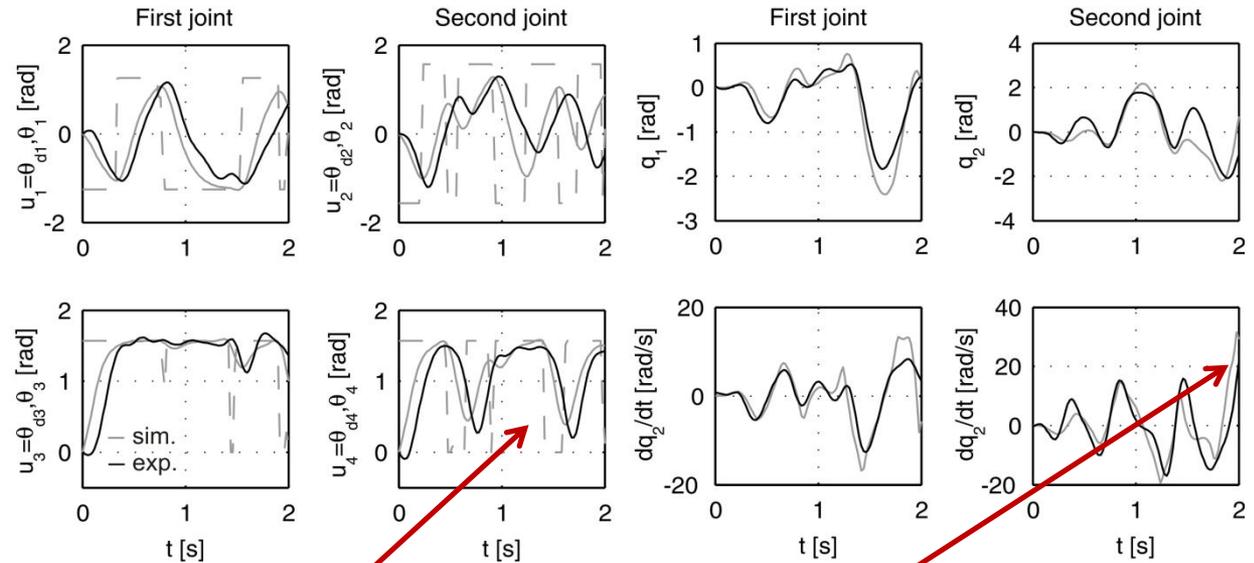
iLQG: Li & Todorov 2007

DDP: Jacobson & Mayne 1970

2-link ball throwing - MACCEPA

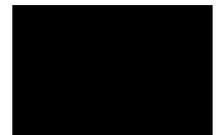


b) Simulated and experimental data



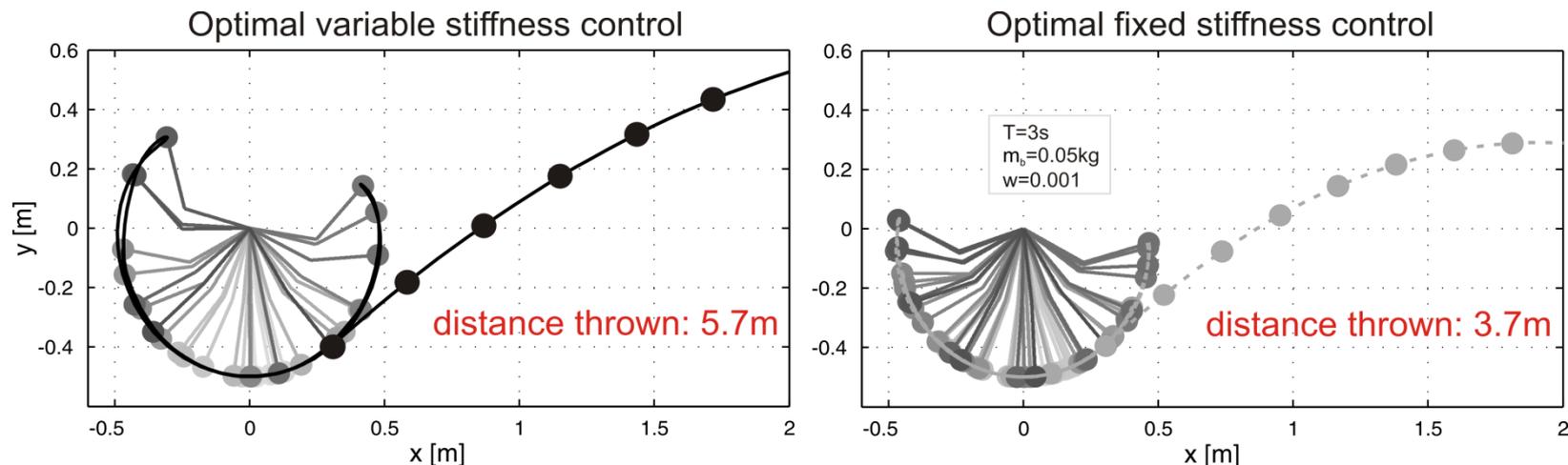
stiffness modulation

speed: 20 rad/s
distance thrown: 5.2m



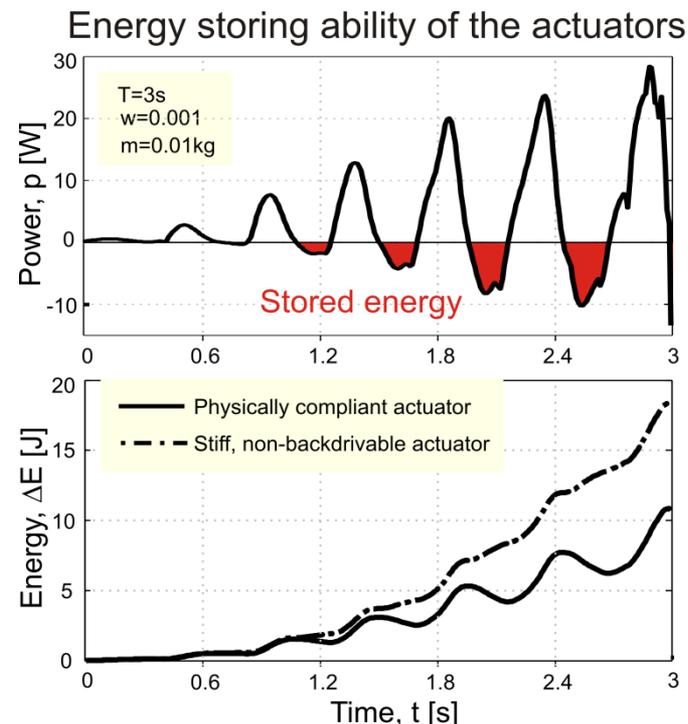
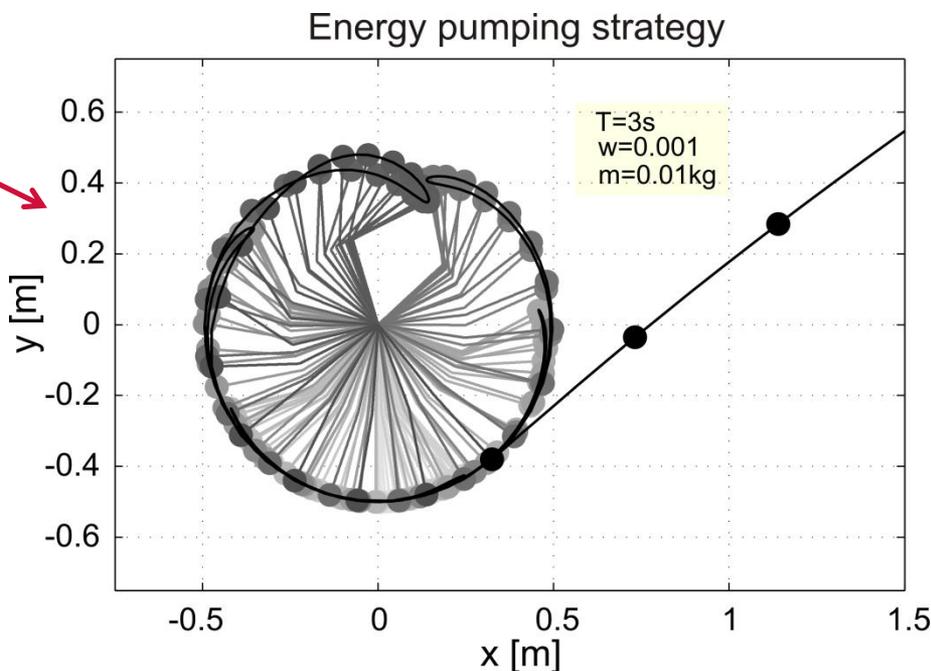
Benefits of Stiffness Modulation:

Quantitative evidence of improved task performance (distance thrown) with temporal **stiffness modulation** as opposed to **fixed** (optimal) stiffness control



Exploiting Natural Dynamics:

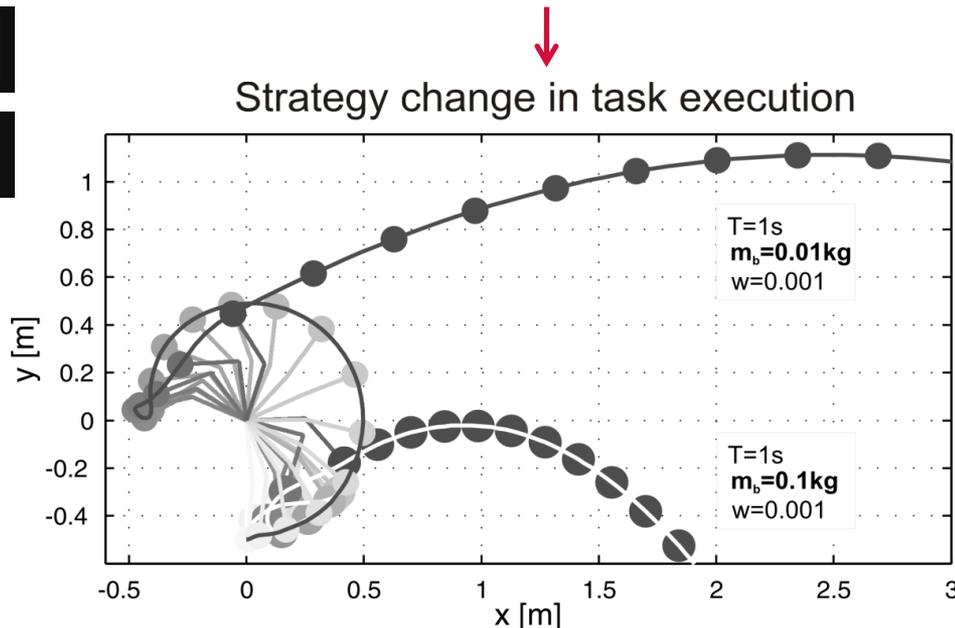
- a) optimization suggests power amplification through pumping energy
- b) benefit of passive stiffness vs. active stiffness control



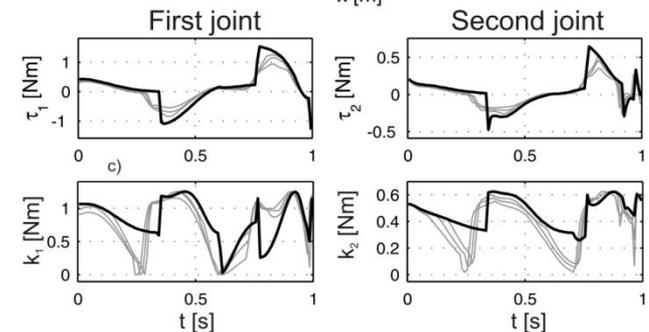
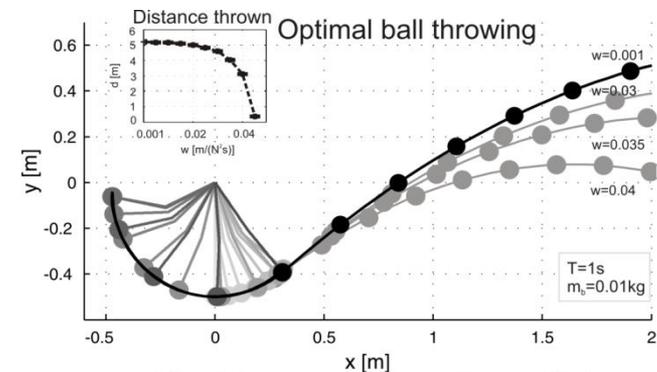
Behaviour Optimization:

Simultaneous stiffness and torque optimization of a VIA actuator that reflects strategies used in human explosive movement tasks:

- a) performance-effort trade-off
- b) qualitatively similar stiffness pattern
- c) strategy change in task execution



$$J = -d + w \frac{1}{2} \int_0^T \|\mathbf{F}\|^2 dt$$



Scalability to More Complex Hardware

DLR HASY:

State-of-the-art research platform for variable stiffness control.

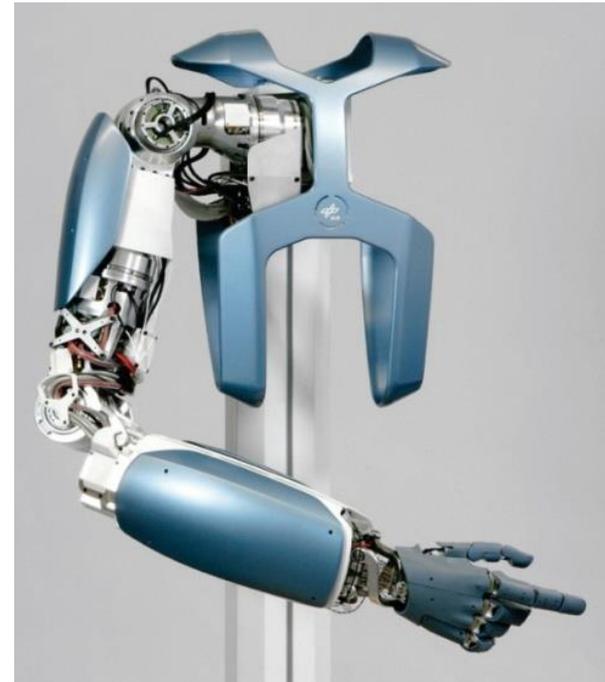
Restricted to a 2-dof system (shoulder and elbow rotation)

Max motor side speed: 8 rad/s

Max torque: 67Nm

Stiffness range: 50 – 800 Nm/rad

Speed for stiffness change: 0.33 s/range



DLR - FSJ

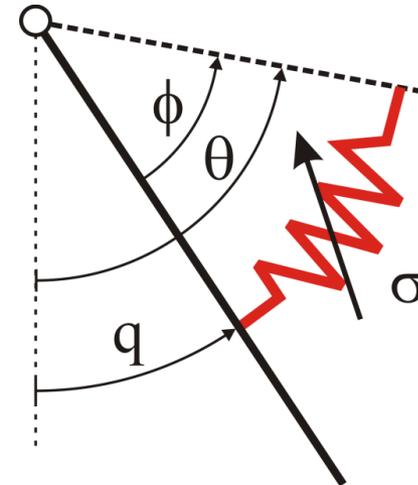
DLR Floating Spring Joint (FSJ)

Adjustable Stiffness Joint

Operating Data			
#	(quantity)	(unit)	(value)
Mechanical			
1	Continuous Output Power	[W]	286.4
2	Nominal Torque	[Nm]	31.3
3	Nominal Speed	[rad/s]	8.51
4	Nominal Stiffness	with no load [s]	0.33
5	with nominal torque [s]	0.33	
6	Peak (Maximum) Torque	[Nm]	67
7	Maximum Speed	[rad/s]	8.51
8	Maximum Stiffness	[Nm/rad]	826
9	Minimum Stiffness	[Nm/rad]	52.4
10	Maximum Elastic Energy	[J]	5.3
11	Maximum Torque Hysteresis	[Nm]	20
12	Maximum deflection	with max. stiffness [°]	3
13	with min. stiffness [°]	15	
14	Active Rotation Angle	[°]	180
15	Angular Resolution	[°]	0.0031
16	Weight	[kg]	1.41
Electrical			
17	Nominal Voltage	[V]	48, 24
18	Nominal Current	[A]	10, 3
19	Maximum Current	[A]	24, 9
Control			
20	Voltage Supply	[V]	12
21	Nominal Current	[A]	1
22	I/O protocol	[]	spacewire

48V 24V 12V Ground Spacewire

Schematic representation of the DLR-FSJ



Motor-side positions:

$$\mathbf{q}_2 = [\theta, \sigma]^T \in \mathbb{R}^4$$

Constraint:

$$\phi_{\min}(\sigma) \leq \phi \leq \phi_{\max}(\sigma)$$

Dealing with Complex Constraints

$$\mathbf{M}_{11}(\mathbf{q}_1)\ddot{\mathbf{q}}_1 + \mathbf{C}_{11}(\mathbf{q}_1, \dot{\mathbf{q}}_1)\dot{\mathbf{q}}_1 + \mathbf{G}_1(\mathbf{q}_1) = \boldsymbol{\tau}_1(\mathbf{q}_1, \mathbf{q}_2)$$

$$\ddot{\mathbf{q}}_2 + 2\beta\dot{\mathbf{q}}_2 + \kappa^2\mathbf{q}_2 = \kappa^2\mathbf{u}$$

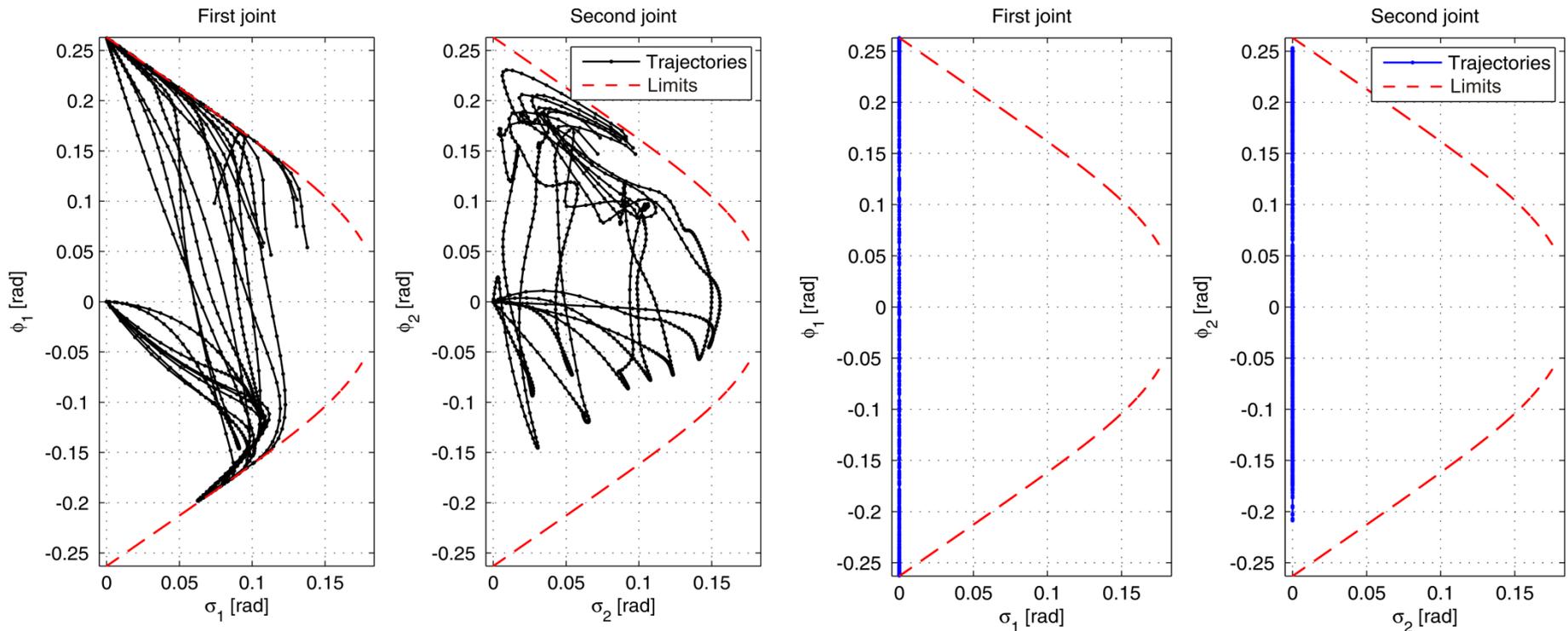
Incorporating the constraints:

1. Range constraints: $\Phi(\mathbf{q}_1, \mathbf{q}_2) \in \Omega = [\Phi_{\min}(\mathbf{q}_2), \Phi_{\max}(\mathbf{q}_2)]$
 $\mathbf{u} \in [\mathbf{u}_{\min}, \mathbf{u}_{\max}] \Rightarrow \Phi(\mathbf{q}_1, \mathbf{q}_2) \in \Omega$
2. Rate/effort limitations: $\kappa \in [0, \kappa_{\max}]$

DLR – FSJ: optimisation with state constraints

variable stiffness

fixed stiffness

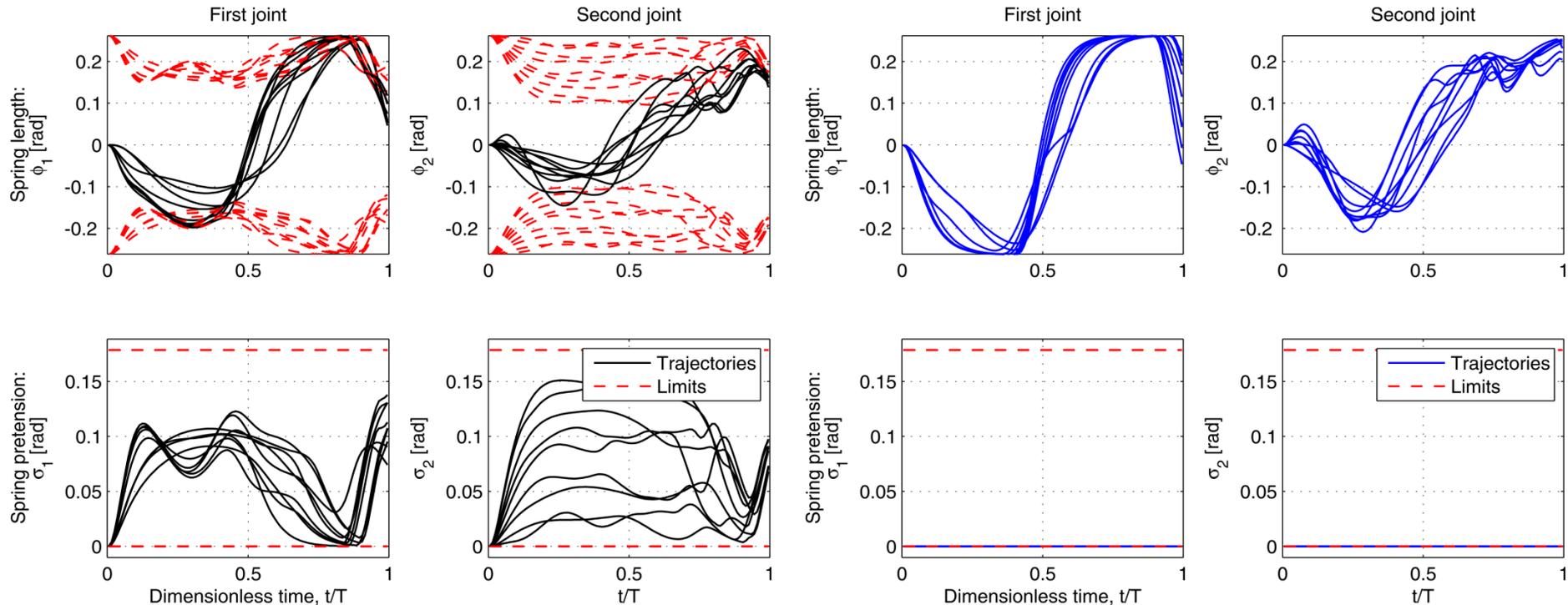


Spring Length vs Stiffness Modulation

DLR – FSJ: optimisation with state constraints

variable stiffness

fixed stiffness



Spring Length and Stiffness Modulation (plotted against time)

Ball throwing with DLR HASy



motor velocity limited to: 2rad/s, 3rad/s

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Periodic Movement Control: Issues

Representation

- what is a suitable representation of periodic movement (trajectories, goal)?

Choice of cost function

- how to design a cost function for periodic movement?

Exploitation of natural dynamics

- how to exploit resonance for energy efficient control?
 - optimize frequency (temporal aspect)
 - stiffness tuning

Cost Function for Periodic Movements

Optimization criterion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

$$J = \Phi(\mathbf{x}_0, \mathbf{x}_T) + \int_0^T r(\mathbf{x}, \mathbf{u}, t) dt$$

Terminal cost

- ensures periodicity of the trajectory

$$\Phi(\mathbf{x}_0, \mathbf{x}_T) = (\mathbf{x}_T - \mathbf{x}_0)^T \mathbf{Q}_T (\mathbf{x}_T - \mathbf{x}_0)$$

Running cost

- tracking performance and control cost

$$r(\mathbf{x}, \mathbf{u}, t) = (\mathbf{x} - \mathbf{x}_{ref})^T \mathbf{Q} (\mathbf{x} - \mathbf{x}_{ref}) + \mathbf{u}^T \mathbf{R} \mathbf{u}$$

$$\mathbf{x} = [y, \dot{y}]^T$$

$$y_{ref}(t) = a_0 + \sum_{n=1}^N (a_n \cos n\omega t + b_n \sin n\omega t)$$

Another View of Cost Function

- Running cost: tracking performance and control cost

$$r(\mathbf{x}, \mathbf{u}, t) = (\mathbf{x} - \mathbf{x}_{ref})^T \mathbf{Q}(\mathbf{x} - \mathbf{x}_{ref}) + \mathbf{u}^T \mathbf{R}\mathbf{u}$$

- Augmented plant dynamics with Fourier series based DMPs

$$\left\{ \begin{array}{l} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} y = r \psi^T(\phi)\boldsymbol{\theta} + y_{offset} \end{array} \right. \quad (2)$$

$$\left\{ \begin{array}{l} \dot{\phi} = \omega \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} \mathbf{z} = \mathbf{x} - \mathbf{y}, \text{ where } \mathbf{y} = [y, \dot{y}] \end{array} \right. \quad (4)$$

- Reformulated running cost

$$r(\mathbf{z}, \mathbf{u}) = \mathbf{z}^T \mathbf{Q}\mathbf{z} + \mathbf{u}^T \mathbf{R}\mathbf{u}$$

- Find control \mathbf{u} and parameter ω such that plant dynamics (1) should behave like (2) and (3) while min. control cost

Temporal Optimization

How do we find the right **temporal duration** in which to optimize a movement ?

Solutions:

- Fix temporal parameters
... not optimal
- Time stationary cost
... cannot deal with sequential tasks, e.g. via points
- Chain '*first exit time*' controllers
... Linear duration cost, not optimal
- **Canonical Time Formulation**

Canonical Time Formulation

Dynamics: $d\mathbf{x} = f(\mathbf{x}, \mathbf{u})\beta dt + g(\mathbf{x}, \mathbf{u})d\eta$

Cost: $J = \sum_{i=1}^N \Phi_i(\mathbf{x}(t_i)) + \int_0^{t_N} [r(\mathbf{x}(t)) + \mathbf{u}(t)^T \mathbf{H} \mathbf{u}(t)] dt$

n.b. t_i represent *real* time

Introduce change of time $t' = \int_0^t \frac{1}{\beta(s)} ds$

Canonical Time Formulation

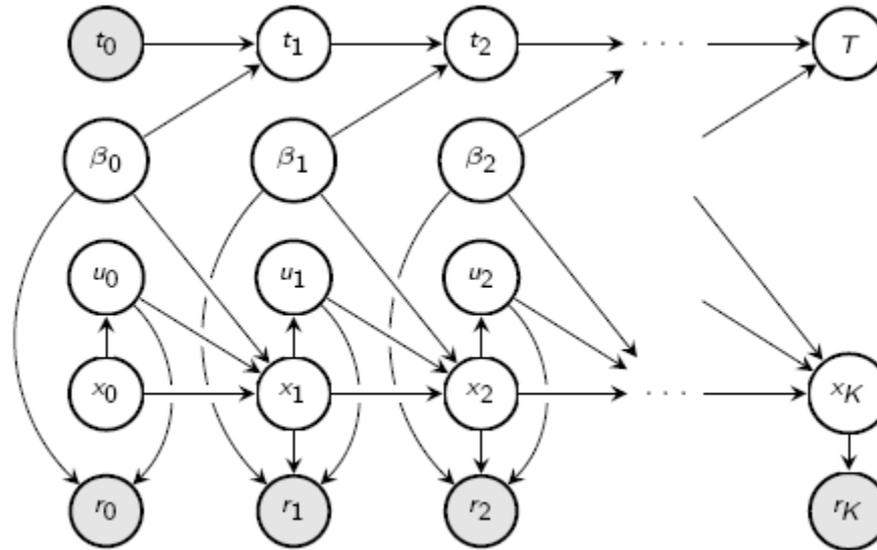
Dynamics: $d\mathbf{x} = f(\mathbf{x}, \mathbf{u})\beta dt' + g(\mathbf{x}, \mathbf{u})d\eta'$

Cost: $J = \sum_{i=1}^N \Phi_i(\mathbf{x}(\tau^{-1}(t'_i))) + \int_0^{\tau^{-1}(t'_N)} [r(\mathbf{x}(t)) + \mathbf{u}(t)^T \mathbf{H} \mathbf{u}(t)] dt$
 $+ \int_0^{t'_N} c(\beta(s)) ds$

n.b. t'_i now represents *canonical* time

Introduce change of time $t' = \int_0^t \frac{1}{\beta(s)} ds$

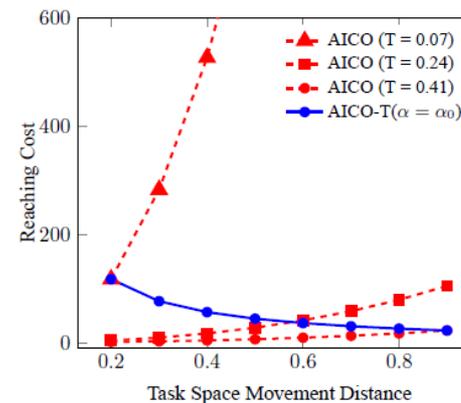
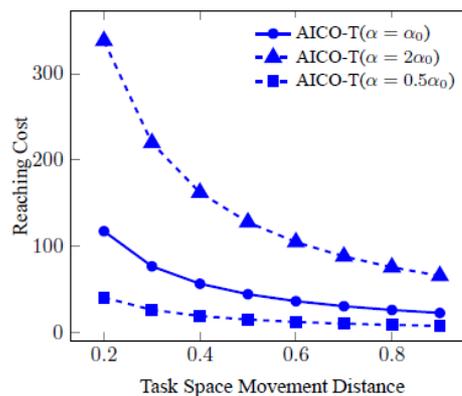
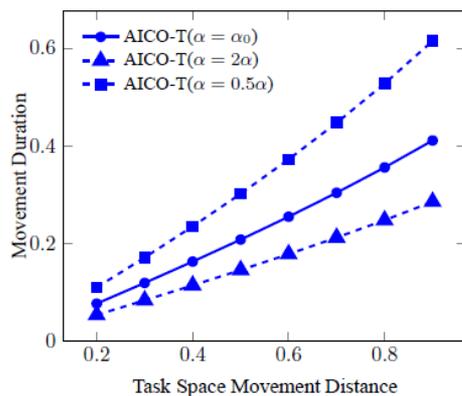
AICO-T algorithm



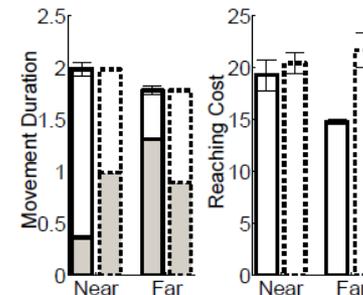
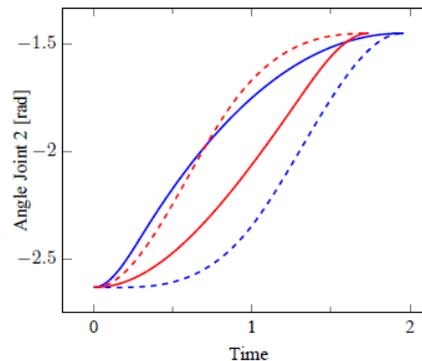
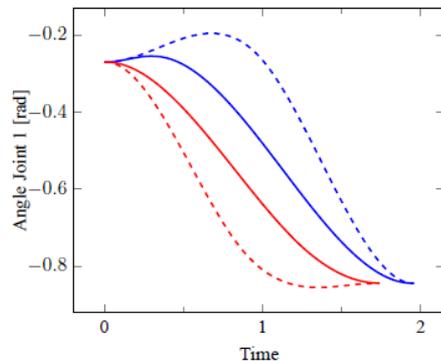
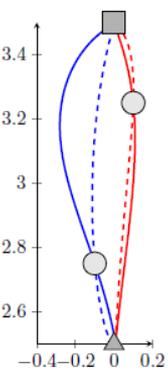
- Use approximate inference methods
- EM algorithm
 - **E-Step:** solve OC problem with fixed β
 - **M-Step:** optimise β with fixed controls

Spatiotemporal Optimization

- 2 DoF arm, reaching task



- 2 DoF arm, via point task



Temporal Optimization in Brachiation

- Optimize the joint torque and movement duration
- Cost function

$$J = (\mathbf{y} - \mathbf{y}^*)^T \mathbf{P}_T (\mathbf{y} - \mathbf{y}^*) + \int_0^T Ru^2 dt$$

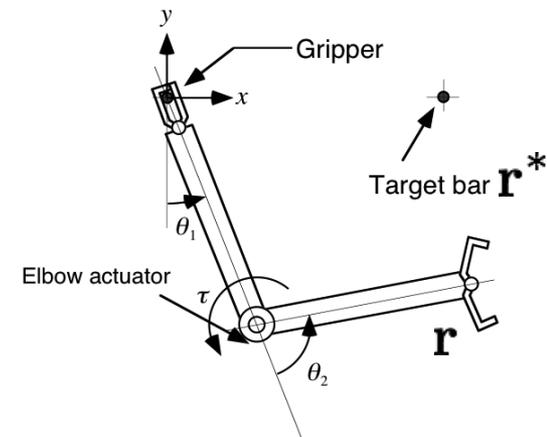
$$\mathbf{y} = [\mathbf{r}, \dot{\mathbf{r}}]^T \in \mathbb{R}^4 \quad \mathbf{r}: \text{gripper position}$$

$$u = \tau$$

- Time-scaling

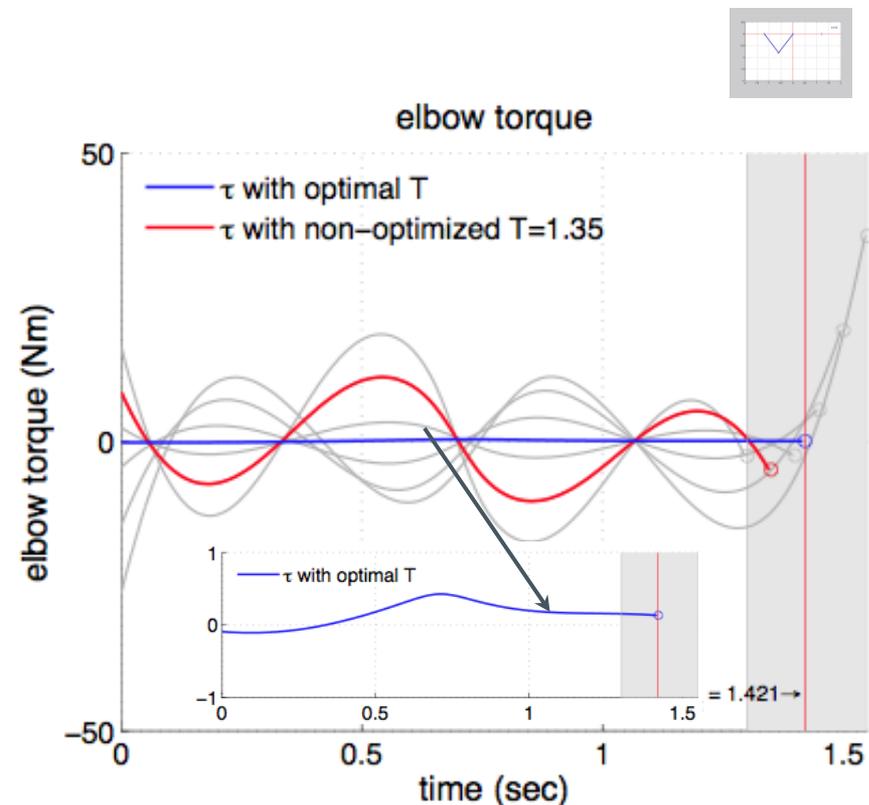
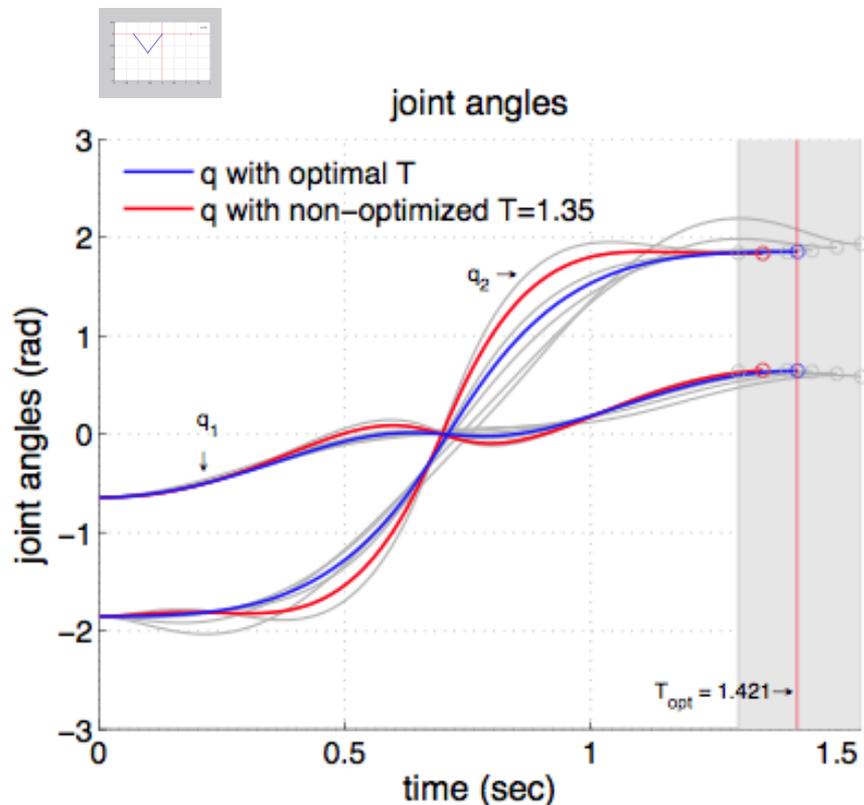
$$t' = \int_0^t \frac{1}{\beta(s)} ds \quad t': \text{canonical time}$$

- Find optimal \mathbf{u}^* using iLQG and update β in turn until convergence [Rawlik, Toussaint and Vijayakumar, 2010]



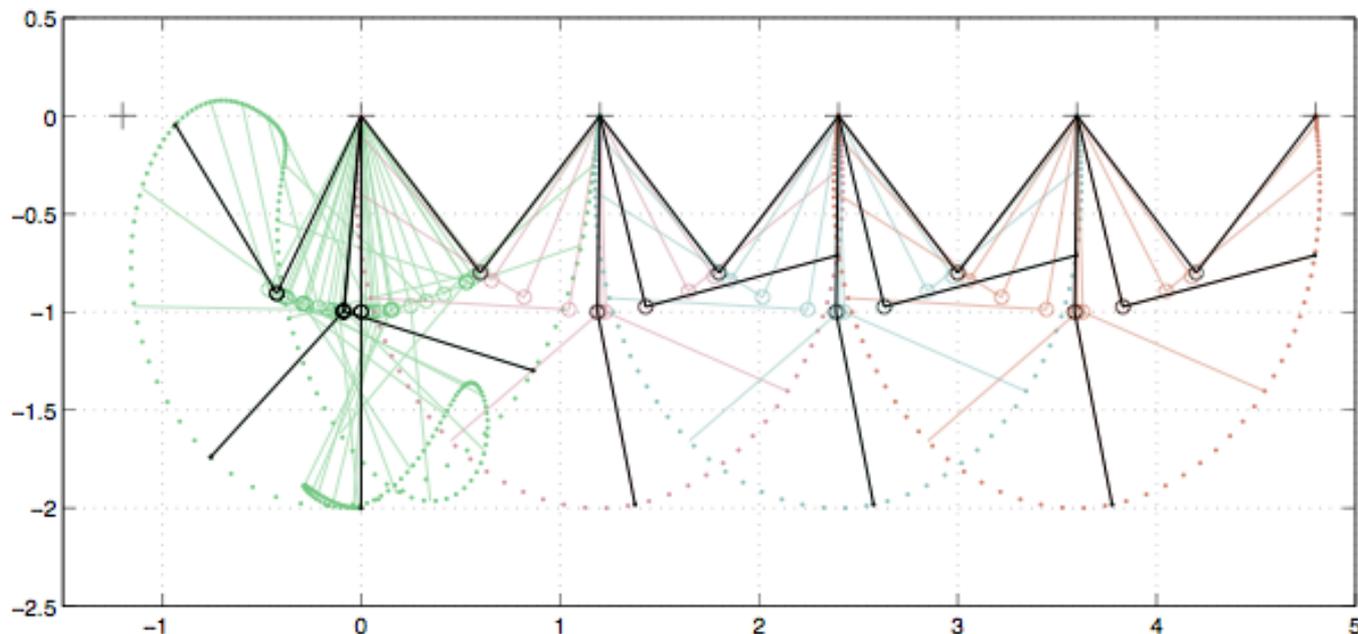
Temporal Optimization of Swing Locomotion

- vary $T=1.3\sim 1.55$ (sec) and compare required joint torque
- significant reduction of joint torque with $T_{opt} = 1.421$

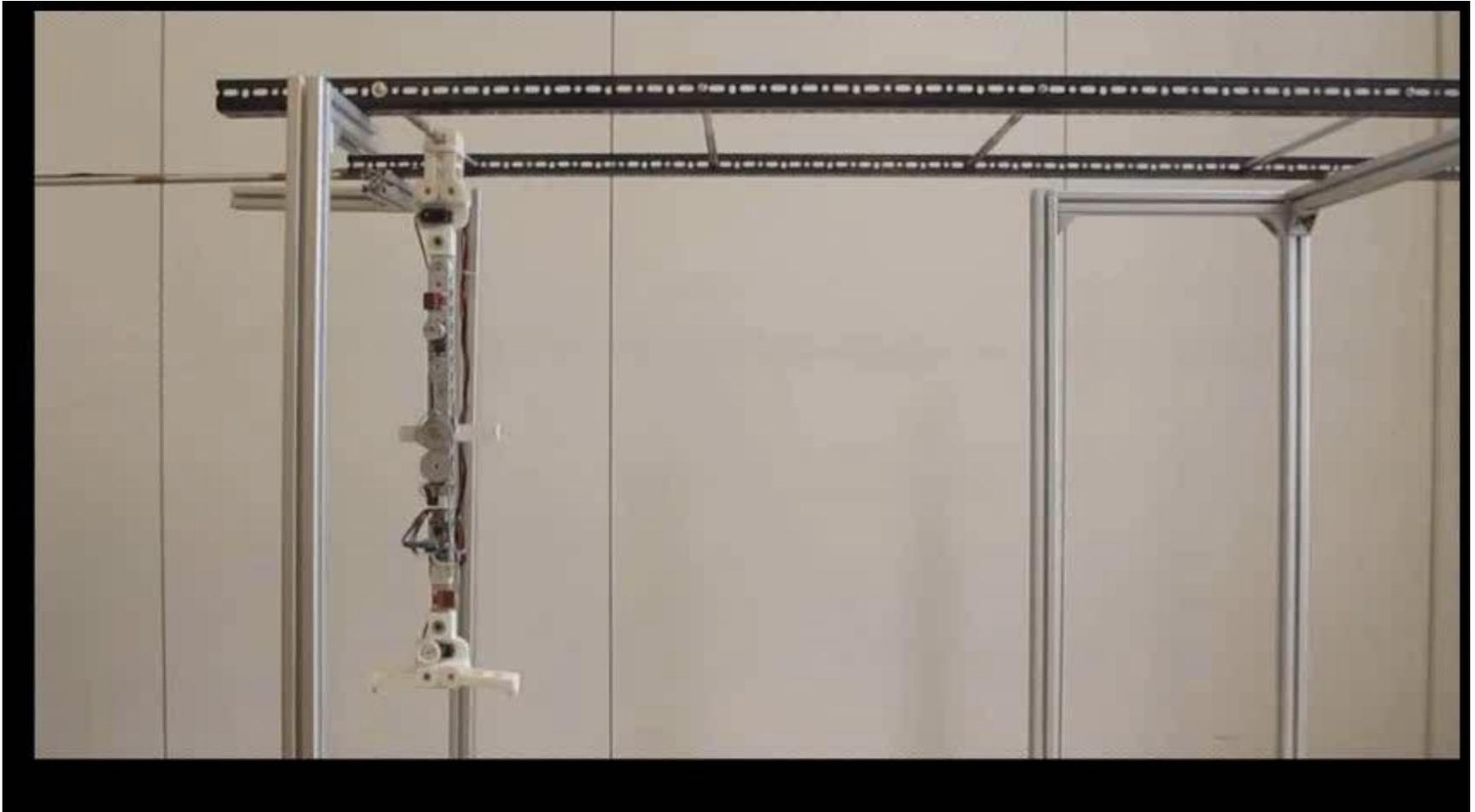


Optimized Brachiating Manoeuvre

Swing-up and locomotion

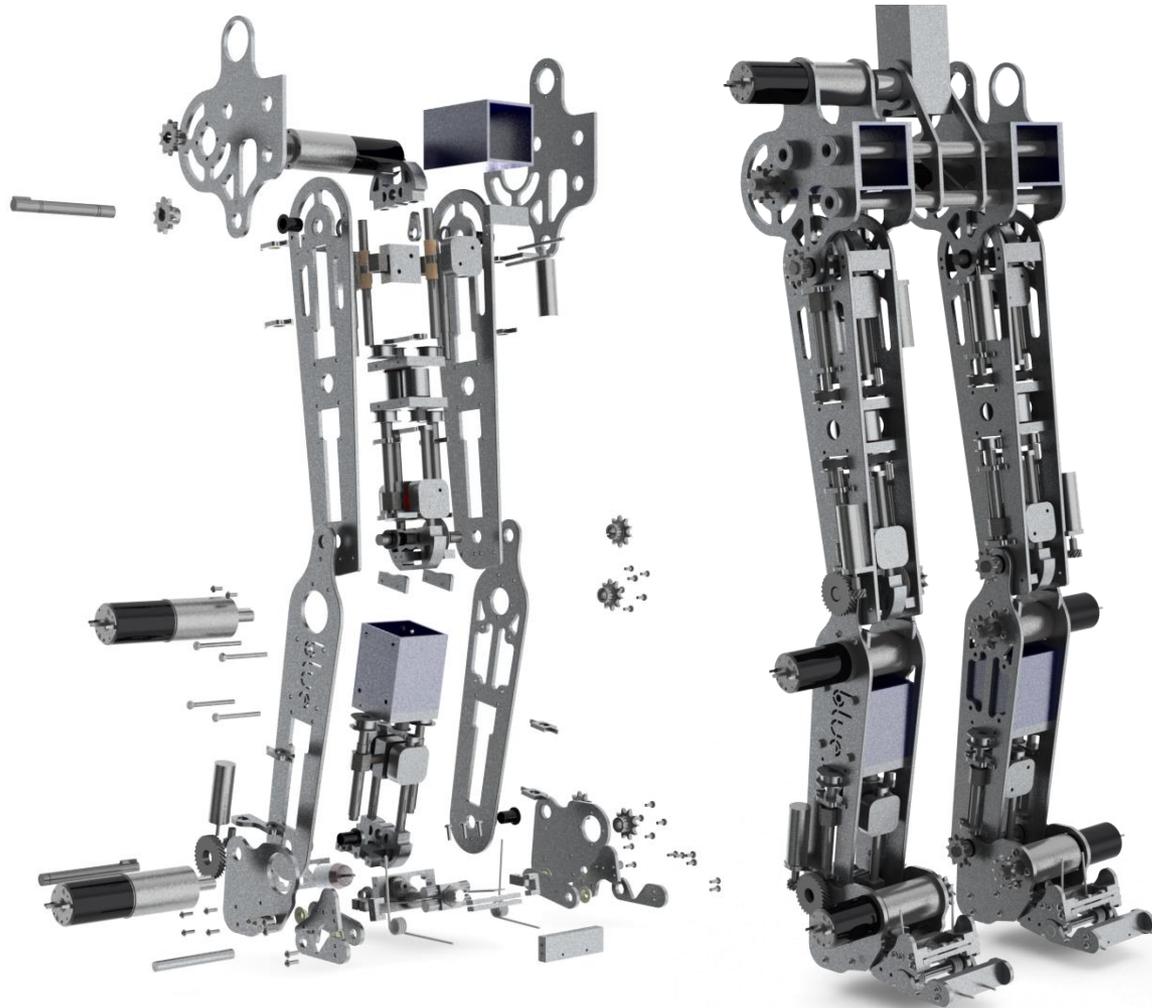


Brachiating Hardware with Constraints



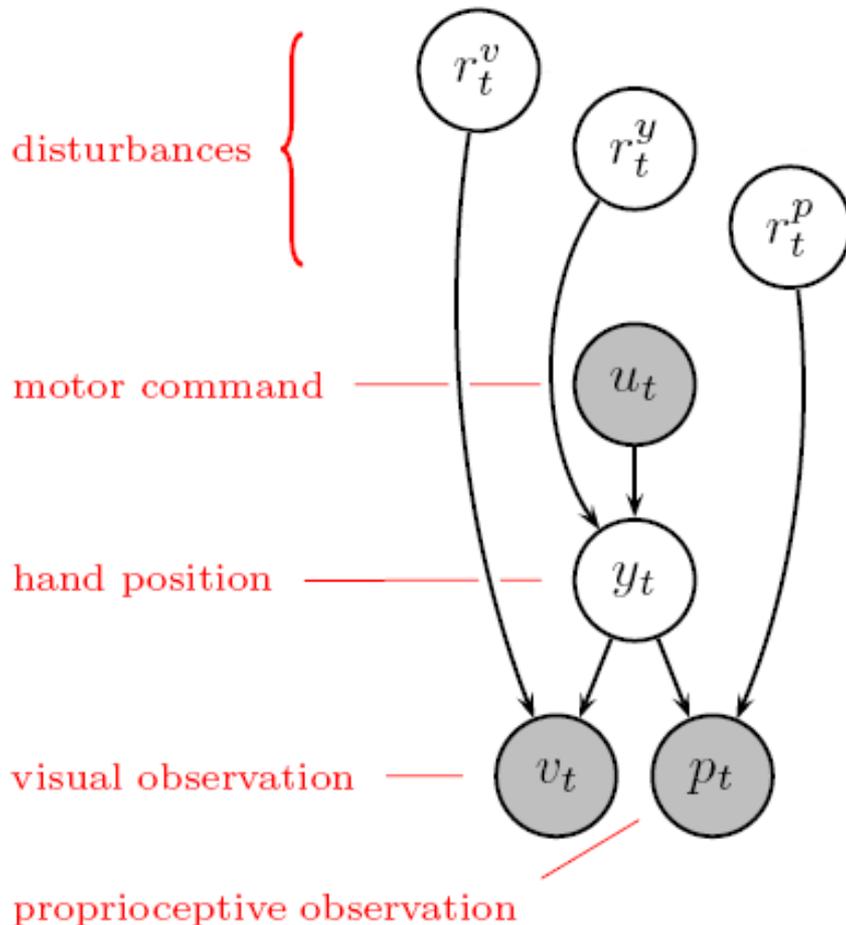


Variable Impedance Biped (BLUE: Bipedal Locomotion @ UoE)



Case Study 1: Causal Modeling of Motor Adaptation

Cue integration under **uncertain causal structure**



- Motor disturbance affects arm position

$$y_t = u_t + r_t + \epsilon_t$$

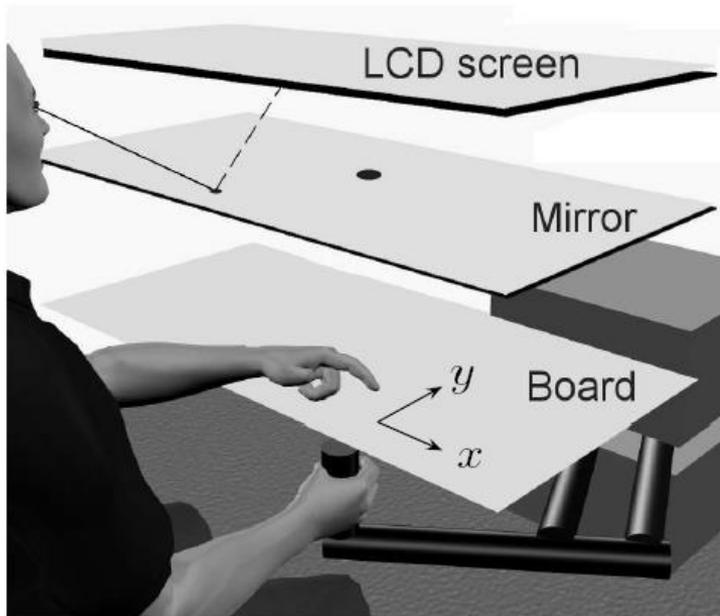
- Sensory disturbances affect observations

$$v_t = y_t + r_t^v + \epsilon_t^v$$

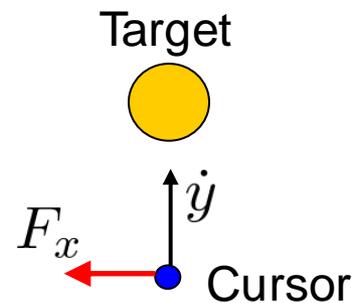
$$p_t = y_t + r_t^p + \epsilon_t^p$$

Theory Driving Novel Experiments

- Test whether force field exposure leads to sensory adaptation
 - Experimental setup and design:



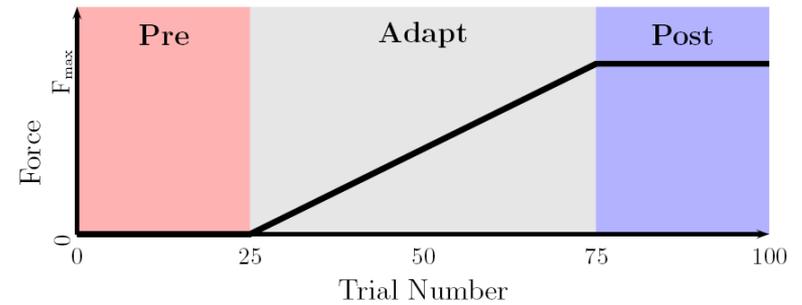
- Reaches in a single direction
- Lateral force applied to hand
 - Forward velocity-dependent



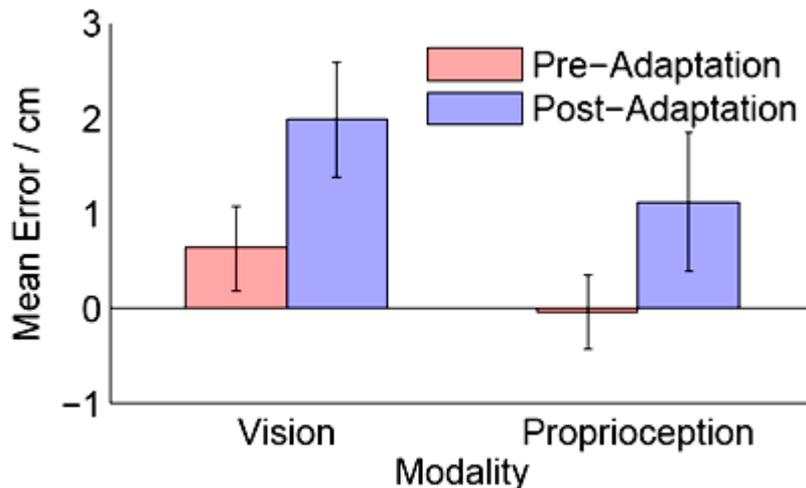
$$F_x = -a \dot{y}$$

Results

- Compare Pre vs Post-adaptation alignment errors



x-direction localization error

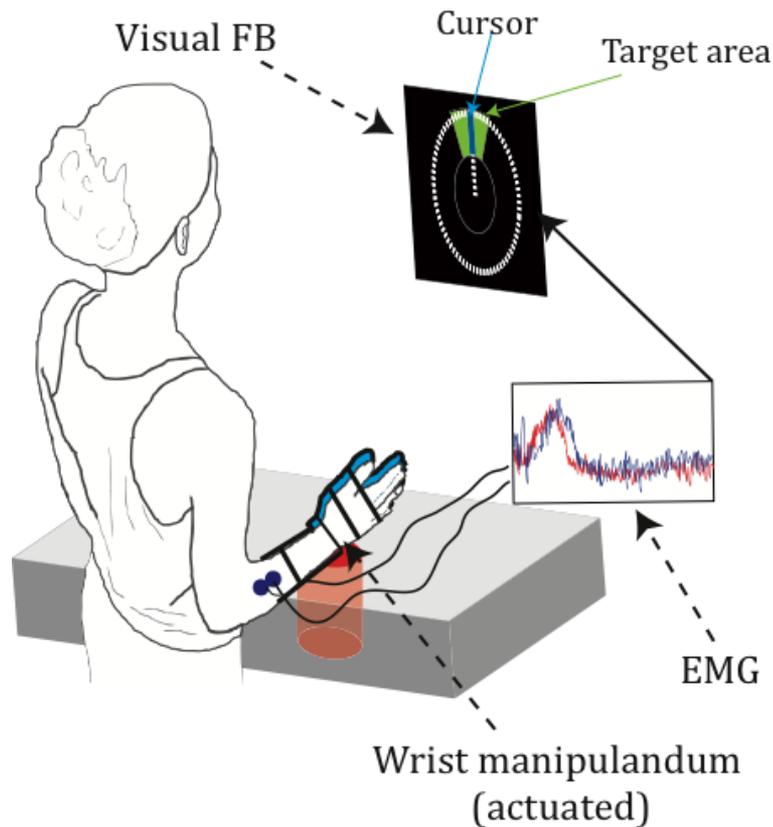


- Significant shifts following adaptation
 - $p < 0.05$ (2-tailed T) for both modalities

Sensory vs Motor Adaptation

- Proves that sensory and motor adaptation are **NOT** independent
 - systematic *motor* perturbation elicits *sensory* recalibration
- Evidence that brain resolves sensorimotor adaptation in a **unified** and **principled** manner
- We should revisit human motor adaptation results/paradigm with this new insight!

Case Study 2: Sensory vs. Motor Noise



- **Does visual perturbation provoke impedance control?**
- Closing the control loop with EMG feedback
- Manipulating visual and proprioceptive feedback
- On-line impedance adaptation and data driven stiffness/visual displacement model

Credits



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 - <http://homepages.inf.ed.ac.uk/svijayak>
- Our group webpage:
 - <http://ipab.inf.ed.ac.uk/slmc>