High-Order Accurate Physical-Constraints-Preserving Schemes for Special Relativistic Hydrodynamics

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Relativistic hydrodynamics (RHD) plays an essential role in many fields of modern physics, e.g. astrophysics. Relativistic flows appear in numerous astrophysical phenomena from stellar to galactic scales, e.g. active galactic nuclei, super-luminal jets, core collapse super-novae, X-ray binaries, pulsars, coalescing neutron stars and black holes, micro-quasars, and gamma ray bursts, etc. The relativistic description of fluid dynamics should be taken into account if the local velocity of the flow is close to the light speed in vacuum or the local internal energy density is comparable (or larger) than the local rest mass density of the fluid. It should also be used whenever matter is influenced by large gravitational potentials, where the Einstein field theory of gravity has to be considered. The dynamics of the relativistic systems requires solving highly nonlinear equations and the analytic treatment of practical problems is extremely difficult. Hence, studying them numerically is the primary approach.

We develop high-order accurate physical-constraints-preserving finite difference WENO schemes for special relativistic hydrodynamical (RHD) equations, built on the local Lax-Friedrich splitting, the WENO reconstruction, the physical-constraints-preserving flux limiter, and the high order strong stability preserving time discretization. They are formal extensions of the existing positivity-preserving finite difference WENO schemes for the non-relativistic Euler equations. However, developing physical-constraints-preserving methods for the RHD system becomes much more difficult than the non-relativistic case because of the strongly coupling between the RHD equations, no explicit expressions of the conservative vector for the primitive variables and the flux vectors, and one more physical constraint for the fluid velocity in addition to the positivity of the rest-mass density and the pressure. The key is to prove the convexity and other properties of the admissible state set and discover a concave function with respect to the conservative vector replacing the pressure which is an important ingredient to enforce the positivity-preserving property for the non-relativistic case.

Several one- and two-dimensional numerical examples are used to demonstrate accuracy, robustness, and effectiveness of the proposed physical-constraints-preserving schemes in solving relativistic problems with large Lorentz factor.