Mathematical and Numerical Analysis of the Time-Dependent Ginzburg-Landau Equations via Hodge Decomposition

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The time-dependent Ginzburg-Landau system models superconductivity in material science. We prove well-posedness of such a system in a non-convex polygonal domain, and decompose the solution as a regular part plus a singular part. We see that the magnetic potential is not in H^1 in general, and a direct use of the standard finite element method may give incorrect solutions. To remedy this situation, we reformulate the equations into an equivalent system of elliptic and parabolic equations based on the Hodge decomposition, which avoids direct calculation of the magnetic potential. The essential unknowns of the reformulated system admit H^1 solutions and can be solved correctly by finite element methods. We then propose a decoupled and linearized algorithm to solve the reformulated equations and present error estimates based on proved regularity of the solution. Numerical examples are provided to support our theoretical analysis and show the efficiency of the method.