Uniform Estimates on Incompressible Surface Waves

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Abstract

We consider the motion of an incompressible viscous fluid, subject to the influence of gravity and surface tension forces, in a moving domain

$$\Omega(t) = \{x \in \mathbb{R}^3 | \quad -b < x_3 < h(t, x_1, x_2)\}.$$

The lower boundary of $\Omega(t)$ is assumed to be rigid and given satisfying a Navier-slip condition, but the upper boundary, the graph of the unknown function $h(t, x_1, x_2)$, is advected with the fluid and satisfies the stress balance dynamical boundary condition. Thus the velocity $u(t, \cdot)$, the pressure $p(t, \cdot)$, and $h(t, x_1, x_2)$ solve the following boundary value problem

$$\begin{array}{ll}
\partial_t u + u \cdot \nabla u + \nabla p - \varepsilon \Delta u = 0 & \text{in} \quad \Omega(t) \\
\nabla \cdot u = 0 & \text{in} \quad \Omega(t) \\
p\vec{n} - 2\varepsilon S(u)\vec{n} = gh\vec{n} - \sigma H \vec{n} & \text{on} \quad \{x_3 = h(t, x_1, x_2)\} \\
\partial_t h = u \cdot \vec{N} & \text{on} \quad \{x_3 = h(t, x_1, x_2)\} \\
u_3 = 0, \ (S(u)(-e_3))_i = -k u_i, \quad i = 1, 2, & \text{on} \quad \{x_3 = -b\}
\end{array}$$

for $S(u) = \frac{1}{2}(\nabla u + \nabla u^t)$, $\vec{n} = \frac{\vec{N}}{|\vec{N}|}$ with $\vec{N} = (-\partial_1 h, -\partial_2 h, 1)$, $\varepsilon > 0$: the viscosity, $g > 0$: gravity, $\sigma > 0$: the surface tension coefficients, k: the friction coefficient, and H is the mean curvature of the free surface. We prove that these exists a uniform time interval on which one can derive uniform estimates independent of both viscosity and surface tension coefficients. These allow one to justify the vanishing viscosity and surface tension limits by the strong compactness argument. As a byproduct, we can get the unified local well-posendess of the free-surface impressible Euler equations with or without surface tension by the inviscid limits. This is a joint work with Dr. Yanjing Wang.