Stress Tensor for a Scalar Field in a Spatially Varying Background Potential: Divergences, “Renormalization”, Anomalies, and Casimir Forces

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Motivated by a desire to understand quantum fluctuation energy densities and stress within a spatially varying dielectric medium, we examine the vacuum expectation value for the stress tensor of a scalar field with arbitrary conformal parameter, in the background of a given potential that depends on only one spatial coordinate. We regulate the expressions by incorporating a temporal-spatial cutoff in the (imaginary) time and transverse-spatial directions. The divergences are captured by the zeroth- and second-order WKB approximations. Then the stress tensor is “renormalized” by omitting the terms that depend on the cutoff. The ambiguities that inevitably arise in this procedure are both duly noted and restricted by imposing certain physical conditions; one result is that the renormalized stress tensor exhibits the expected trace anomaly. The renormalized stress tensor exhibits no pressure anomaly, in that the principle of virtual work is satisfied for motions in a transverse direction. We then consider a potential that defines a wall, a one-dimensional potential, that vanishes for $z < 0$ and rises like z^a, $a > 0$, for $z > 0$. Previously, the energy density had been computed to the left of the wall, whereas now we compute all components of the stress tensor in the interior of the wall. The full finite stress tensor is computed numerically for the two cases where explicit solutions to the differential equation are available, $a = 1$ and 2. The energy density exhibits an inverse linear divergence as the boundary is approached from the inside for a linear potential, and a logarithmic divergence for a quadratic potential. Finally, the interaction between two such walls is computed, and it is shown that the attractive Casimir pressure between the two walls also satisfies the principle of virtual work (i.e., the pressure equals the negative derivative of the energy with respect to the distance between the walls).