Approximate Separability of Green's Function for Helmholtz Equation in the High Frequency Limit

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Approximate separable representations of Green's functions for differential operators is a basic and important question in analysis of differential equations and development of efficient numerical algorithms. It also reveals the intrinsic complexity or degrees of freedom of the problem. Being able to approximate a Green's function as a sum with few separable terms is equivalent to the existence of low rank approximation of the corresponding discretized operators which can be explored for matrix compression and fast solution techniques. Green's function for coercive elliptic differential operator has been shown to be highly separable. However, the case of Helmholtz equation in the high frequency limit is more challenging both mathematically and numerically. We develop a new approach to study approximate separability for the Green's function of Helmholtz equation in the high frequency limit based on an explicit characterization of the relation between two Green's functions and a tight dimension estimate for the best linear subspace approximating a set of almost orthogonal vectors. We derive both lower bounds and upper bounds and show their sharpness and implications for computation setups that are commonly used in practice.

This is a joint work with Bjorn Engquist.