A Multiscale Model Reduction Method for Partial Differential Equations

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We propose a multiscale model reduction method for partial differential equations. The main purpose of this method is to derive an effective equation for multiscale problems without scale separation. An essential ingredient of our method is to decompose the harmonic coordinates into a smooth part and a highly oscillatory part so that the smooth part is invertible and the highly oscillatory part is small. Such decomposition plays a key role in our construction of the effective equation. We show that the solution to the effective equation is in H2, and can be approximated by a regular coarse mesh. When the multiscale problem has scale separation and a periodic structure, our method recovers the traditional homogenized equation. Furthermore, we provide error analysis for our method and show that the solution to the effective equation is close to the original multiscale solution in the H1 norm. Numerical results are presented to demonstrate the accuracy and robustness of the proposed method for several multiscale problems without scale separation, including a problem with a high contrast coefficient.